BRL

REPORT NO. 1106
AUGUST 1960

PHOTOGRAMMETRIC EVALUATION OF AIRCRAFT GUN FIRINGS AT HIGH ALTITUDES

Myron W. Lawrence

Department of the Army Project No. 5B03-06-011
Ordnance Management Structure Code No. 5210.11.143

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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(Ordnance Research and Development Project No. TB3-0538)

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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ABSTRACT

This report describes methods of measurement and reduction of data on aircraft gun firings at high altitudes.

These methods were applied to records obtained by airborne and ground based photogrammetric cameras for the purpose of determining:

(a) the spatial orientation of the gun at the moment of firing,
(b) the spatial coordinates and velocity of the aircraft at the time of firing,
(c) the muzzle velocity and the spatial coordinates of the point of burst of the projectile.

Results of several rounds are tabulated and numerical examples of each type of computation are included in this report.

A detailed description of the instrumentation is contained in BRL Report No. 1105, "Instrumentation for Acquisition of Ballistic Data on Full-Scale Aircraft Gun Firings", by Donald F. Menne, August 1960.

1. INTRODUCTION

1.1 In January 1954, the Ballistic Research Laboratories were assigned responsibility for developing instrumentation, methods of observation, and data reduction procedures for the acquisition of ballistic data on aircraft gun firings at high altitudes. This assignment comprising the full scale phase of project "Crosswind" was discussed in an earlier report.(1)

The Ballistic Research Laboratories were responsible for the first group of firings. Range support and additional personnel were provided by the Air Force Armament Center (AFAC), Eglin Air Force Base, Florida. The field observations during the initial phase of this program were conducted at Eglin Air Force Base (EAFB) between February and June 1958. The data reductions were carried out by BRL at the Aberdeen Proving Ground, Maryland. The BRL survey group departed EAFB in March 1958, and the remaining BRL personnel in May 1958. Upon completion of this initial phase, all instrumentation and data were turned over to AFAC.

1.2 The quantities to be determined, their respective accuracies and method of observation, as set forth in Reference (1) were as follows:

(a) Spatial coordinates of the aircraft at the instant the gun is fired and of the burst of the projectile to a relative accuracy of $\pm$ 2 ft. (mean error), by ground based phototheodolites

(b) Velocity of the aircraft to $\pm$ 2 ft/sec*, by ground based phototheodolites

(c) Orientation of the gun line in space to $\pm$ 1 mil, by airborne photogrammetric camera

(d) Muzzle velocity of the projectile to $\pm$ 2 ft/sec, by velocity cameras, as well as by electronic projectile velocity meter (PVM)

(e) Time of emergence of the projectile from the muzzle to $\pm$ 0.001 sec, by eletronic method

(f) Time of burst of the projectile to $\pm$ 0.001 sec, by electronic method

*This value replaces the original requirement of 1 ft/sec by mutual agreement between BRL & AFAC.

(g) Meterological data including:

(1) Air density to $\pm$ 0.1%

(2) Wind velocity to $\pm$ 5 ft/sec.

(3) Air temperature to $\pm$ 1^{o}F

This report will be concerned only with items (a) through (c), and with (d) insofar as the optical method is concerned. Table 27 lists a summary of results of these items together with results as obtained by electronic methods. Item (g) which was the responsibility of AFAC is only included insofar as these data are used to compute refraction corrections.

1.3 An F-89C aircraft (Fig. 1), modified to carry the required instrumentation (Fig. 2), was used. All missions were flown at 30,000 feet, where 20mm M97 rounds using gun M-24 were fired. All firings were made approximately vertically downward.

1.4 This report deals with a group of 15 rounds fired at EAFB. In the case of some firings it was impossible to obtain all the desired quantities because of failures of one type or another. However, all data that could be reduced for these rounds have been included.

Throughout this report the rounds are designated by date, mission, pass and round number. For example, 11 April M1/P3-2 refers to the 2nd round fired during the 3rd pass over the range for the 1st mission flown on the 11th of April. If only one round was fired during any one pass the round number was omitted.

1.5 Measurements of plate and film coordinates were made on a precision Mann Comparator where readings to the nearest micron were taken. In the case of the airborne camera (Wild RC-7) plates and velocity camera film, three settings were made on each point measured. In the case of the ground based phototheodolite (Askania) plates, three settings were made on each star break resulting in twelve settings for each star. Three settings were made on each survey flash image and twelve settings on the burst image. In all cases, six settings were made on each fiducial mark, three before and three after the plate or film was measured.

Fig. 1 F-89 Aircraft

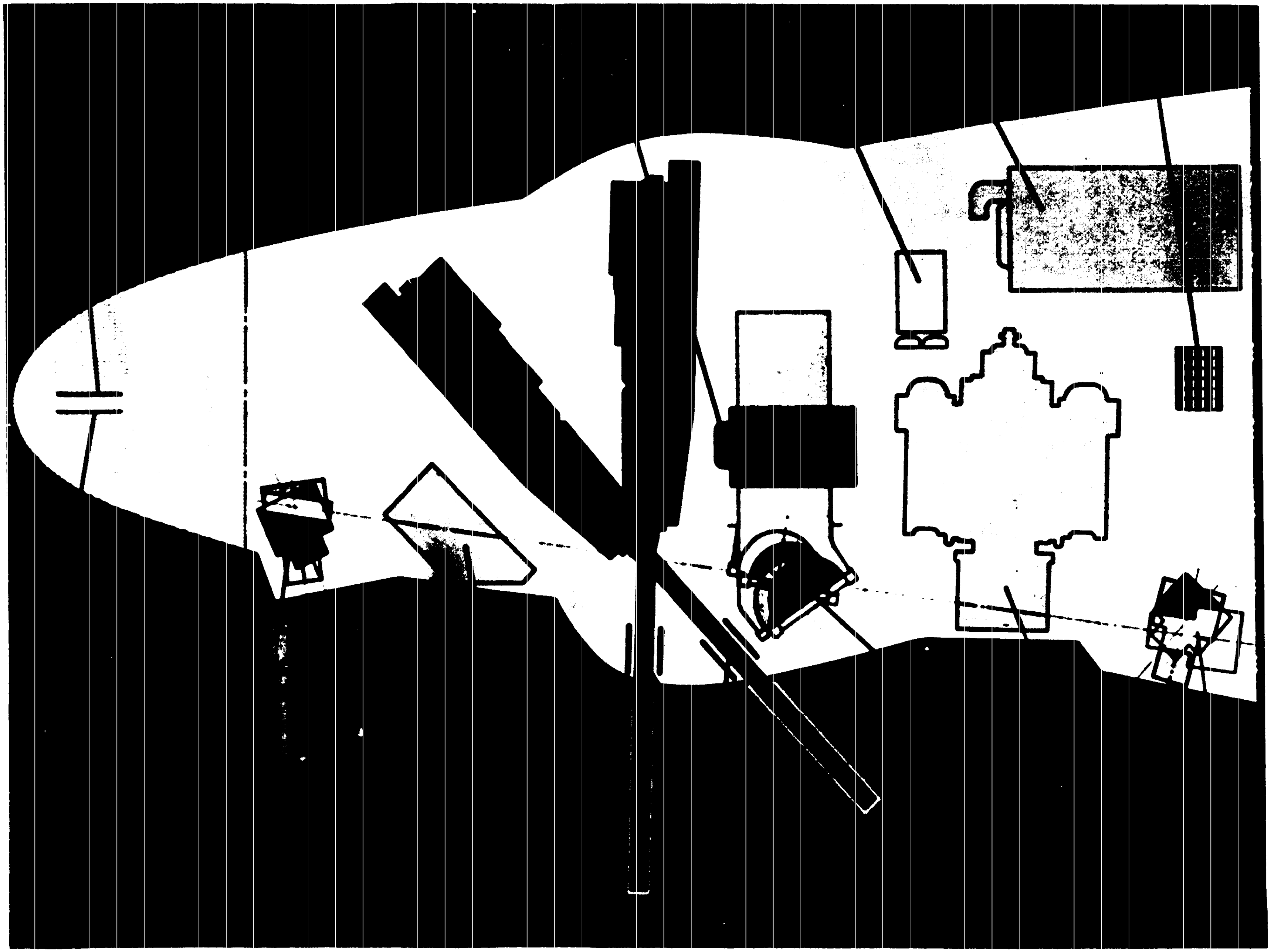


Fig. 2 Airborne Instrumentation

1.6 The ORDVAC and EDVAC high speed electronic computers were available for the computation of star positions, range and calibration pit control point coordinates and the reduction of camera orientations and gun directions.

1.7 BRL Report No. 1105, "Instrumentation for Acquisition of Ballistic Data on Full-Scale Aircraft Gun Firings"(2) gives a detailed description of the instrumentation system.

2. CALIBRATION PIT

2.1 Function and Description

The basic function of the calibration pit was to determine the direction of the gun relative to the RC-7 camera and the orientations of the velocity cameras, by means of calibration targets, photographed from the stationary aircraft.

The calibration pit (Fig. 3), was constructed of reinforced concrete, approximately 23 feet square and 13 feet deep. At the top and flush with the natural ground surface a cantilever type beam, 2 feet wide and extending 7 feet over the pit, was constructed on which the front wheel of the aircraft could be placed. This allowed positioning of the RC-7, the velocity cameras and the gun over the pit.

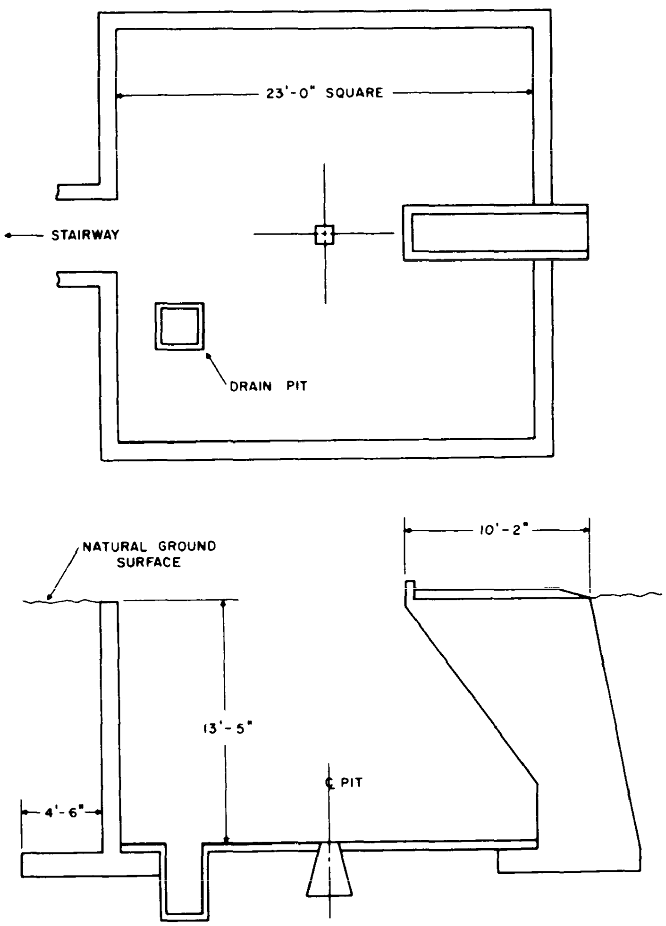


FIG 3 - CALIBRATION PIT

To provide the targets or control points necessary to determine the RC-7 camera orientation 25 steel balls, each .06 inches in diameter and painted white, were attached to an invar bar system (Fig. 4). This system, mounted to an isolated pier in the center of the bottom of the calibration pit, consisted of eight separate invar bars radiating from the center toward the four walls of the pit. Two control points (Nos. 26 and 27) were mounted on a rod which was temporarily inserted into the gun barrel to define its center line during the calibration. Control point No. 25 was used to determine the camera orientations for velocity cameras Nos. 1 and 4. Control point No. 27 was used to determine the camera orientations for velocity cameras Nos. 2 and 3.

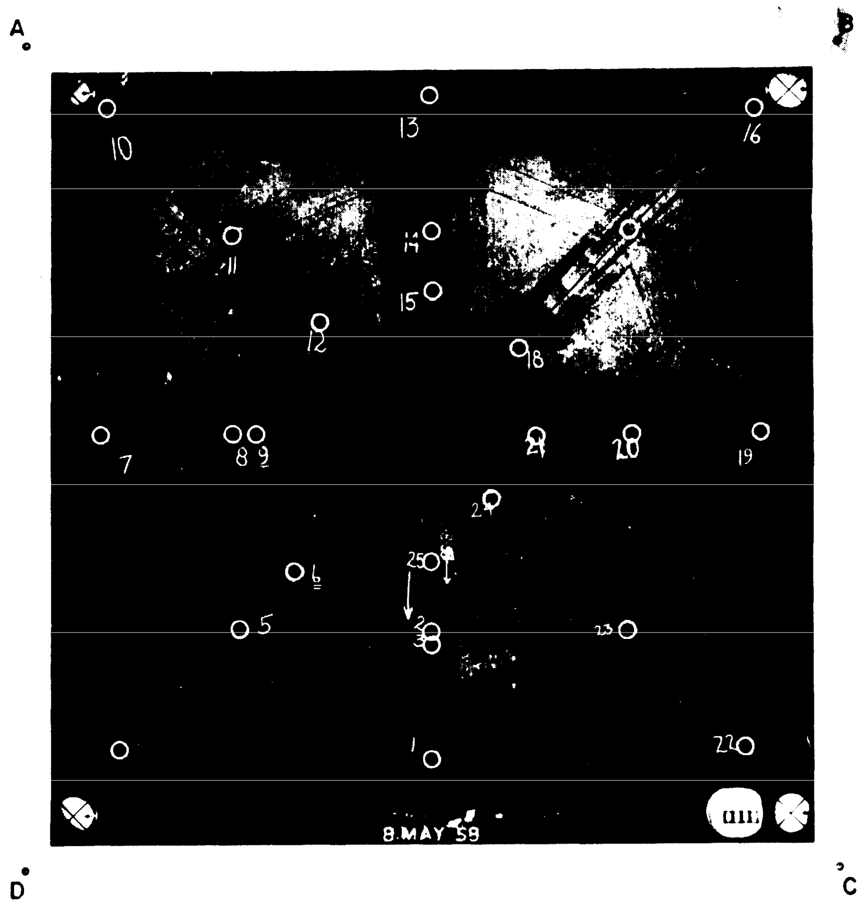


Fig. 4 RC-7 Photograph of Calibration Pit

At the top of each corner of the pit, permanent type self-centering bolts were embedded in the concrete, thus allowing the positions of Wild T-2 theodolites to be recovered in all three coordinates. A calibrated invar bar 10.5 feet long which could be used to determine the scale of the control points system was attached to one wall of the pit. (Scale was actually determined by taping the distance between two self-centering bolts and checked by the invar bar).

Except for the glossy white target balls, which were illuminated by 16 evenly spaced 500 watt photo-flood type bulbs, the calibration pit and invar bar system were painted a dull non-reflecting black.

2.2 Pit Calibrations

As a result of numerous pit calibrations, a procedure for observing the pit control points was established.

(a) Periodically, a complete pit calibration was made in which all 25 control points and stations A, B, C and D were observed. Coordinates determined from this survey were used to determine the RC-7 camera orientation. During this calibration, a Wild T-2 theodolite was set up at each corner (A, B, C, D) of the pit. A quadrilateral was thereby formed from which pointings were made to and from each theodolite position. To determine the XY-coordinates of the 25 control points, this quadrilateral (Fig. 5) was adjusted and served as a basic net from which all control points were cut in by intersection. Because of the steep vertical angles encountered in the pit, only the six control points, Nos. 7, 8, 9, 19, 20 and 21 were observed from all four stations. Control points Nos. 10 through 18 were observed from stations D and C, and control points Nos. 1 through 6 and 22 through 25 were observed from stations A and B. (Fig. 4). The Z coordinates were determined by differential leveling.

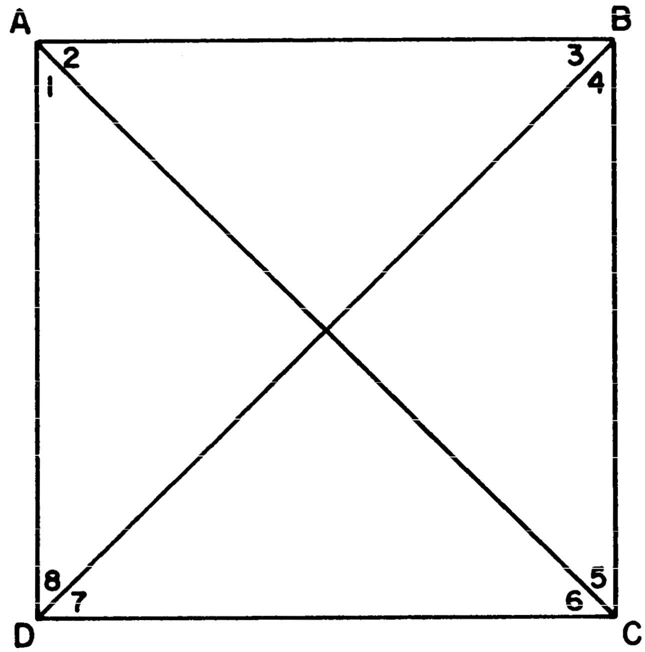


FIG. 5 – CALIBRATION PIT QUADRILATERAL

(b) A check was made on the relative positions of the control points before each night's missions. Six points, Nos. 9, 15, 17, 19, 23 and 25 were chosen for this purpose. The Z coordinates were checked by vertical angles turned from station D and XY coordinates by intersection from stations A and D. Any changes in their relative positions large enough to affect the camera orientations would require a new pit calibration.

During the times the check survey was made, the aircraft was positioned over the pit, control points Nos. 26 and 27 were observed and velocity and RC-7 camera photographs were taken. The relations among velocity cameras, RC-7 camera, and the gun could in this way be established.

2.3 Stability of the Control Points

The stability of the control points depends on the stability of the invar bar system and the concrete pit itself. (See Table 1 which shows the XYZ coordinates of 6 of the 25 control points as obtained from different pit calibrations.)

Pit calibrations made during the summer of 1957 showed that the invar bar system was stable and that most of the changes in the coordinates of the control points were due to deflections or changes in the walls of the pit. These changes would affect the quadrilateral or basic net from which the coordinates of the control points were determined (See Tables 2, 3, 4 and note the changes in the adjusted angles, the four coordinates and the six lengths of the quadrilateral.) Changes due to the instability of the quadrilateral would not affect camera orientations or the direction of the gun; as long as the relative difference in the coordinates of the control points remained the same, the relations among velocity cameras, RC-7 camera and the direction of the gun can be determined.

2.4 Accuracy of the Control Points

The accuracy with which the positions of the control points could be determined depended on (a) the length measurements (scale), (b) the differential levels or vertical angles, and (c) the horizontal angles to the stations and control points.

(a) Scale

Scale would affect the absolute accuracy but was important only for points Nos. 25 and 27. As these two points were indirectly used to determine the projectile velocity, a true distance between them was necessary. Since only directions were required for the RC-7 camera and the gun, any scale could have been used.

Scale comparisons made on the 10.5 foot invar bar showed errors to be less than 0.2mm. Assuming an error of approximately 0.2mm for the distance between points Nos. 25 and 27, and taking 0.004 seconds as the

average time required for the projectile to travel a distance equal to the distance between the two points, an error for the projectile velocity of 0.05 meters per second would result. The effect of scale errors of the calibration pit on the projectile velocity was, therefore, negligible.

(b) Differential levels and vertical angles.

To determine the elevations of the control points, two Wild N-2 instruments were set up in the bottom of the pit at opposite corners, because the minimum focal distance of the instruments would not accommodate all the points from one set-up. Elevations of the control points that could be observed from both set-ups agreed to within 0.2mm. Checks on the elevations made by changing the instrument heights and re-running the points were within 0.2mm.

Vertical angles to the control points (see 2.2(b)), which were used to check the Z coordinates, could easily be turned to within 5 seconds or arc. The maximum oblique distance from an instrument to a control point, station D to point 16 was approximately 9.6 m. The Z coordinates of the control points, as determined by vertical angles, should therefore be good to 0.2mm. (See Table 5 which is a tabulation of Z coordinates as obtained from several calibrations). The two calibrations of 28 February 1958, Nos. 07 and 08, were made approximately 5 hours apart. Calibration Nos. 05, 19, 22, 24, and 36 were complete pit calibrations, the Z coordinates of which were obtained by differential levels, whereas all others were obtained by vertical angles. It is noted that the differences in coordinates on the 07 and 08 runs were within 0.2mm, and also that runs 05 and 09 determined by levels and vertical angles were in good agreement. In fact all runs from 28 February to 11 March would indicate that the Z coordinates of the control points not only were stable but also were within the expected limits of accuracy. Calibrations run from 21 March to 5 June were fair but showed a certain amount of relaxation in attitude towards accuracy on the part of the observer. These results could only be attributed to coarse errors in the observations or incorrect instrument heights. The permanent

type self-centering bolts provided a means of not only positioning the theodolite but also of establishing the same instrument heights for all calibrations. Some of the calibrations, for example calibration No. 15, (see Table 5) seemed to be consistently off by approximately 2.5mm. This discrepancy could be the result of not setting the instrument to the correct height, since careful set-ups should position the instrument in height to about 0.1mm. Inasmuch as the pit control points would directly affect the camera calibrations, maximum care in establishing coordinates of the control points should be exercised.

The two control points on the gun rod also afforded a check on the accuracy that could be expected from a calibration. These two points, Nos. 26 and 27, used to define the gun center line, were surveyed in by observing horizontal angles from Stations A and D and vertical angles from Station D. In this way the XYZ coordinates of the two control points could be computed. Once the coordinates were known, the required direction of the gun could be determined and the measured distance between the two control points used as a check. This distance, measured by taping to be 0.8522 m, remained sufficiently constant. Table 6 is a tabulation of this distance computed from the XYZ coordinates of the two points. The results between 11 February and 21 May compared favorably with the measured distance. The sudden breakdown of this agreement, as shown by results between 22 May and 4 June, could be attributed to several factors, one being careless field observations, another being a movement by the aircraft or the gun during the time points Nos. 26 and 27 were being observed. An adjustment of calibrations made between 11 February and 21 May resulted in a mean error for an individual distance measurement of $\pm$ 0.18mm. Therefore, the mean error of the coordinates of a single point would be:

$$\pm \frac{0.18}{\sqrt{2}} = \pm 0.13\text{mm}$$

This was somewhat better than the accuracy obtained for the 25 control points in the pit and was probably due to the smaller vertical angles encountered when observing points Nos. 26 and 27. The vertical angles to points Nos. 26 and 27 were approximately + $1^{\circ}\ 40'$ and $-9^{\circ}\ 50'$ whereas vertical angles to the control points in the pit ranged from $-45^{\circ}\ 50'$ for point No. 25 to $-29^{\circ}\ 10'$ for point No. 17. These large vertical angles, unless corrected for by bubble readings or by carefully leveling the horizontal axis of the instrument, would affect the horizontal directions to the points. During the calibration the leveling method was used.

(c) Horizontal angles

A method for determining the XY coordinates and mean errors of the pit control points by the method of least squares will be shown in a following example. Part A is the adjustment of the quadrilateral ABCD, Part B the adjustment of the four directions to one of the six over-determined control points (see Tables 7, 8 and 9, which give the results of Parts A and B for pit calibrations run on 3 March, 22 April and 6 May respectively).

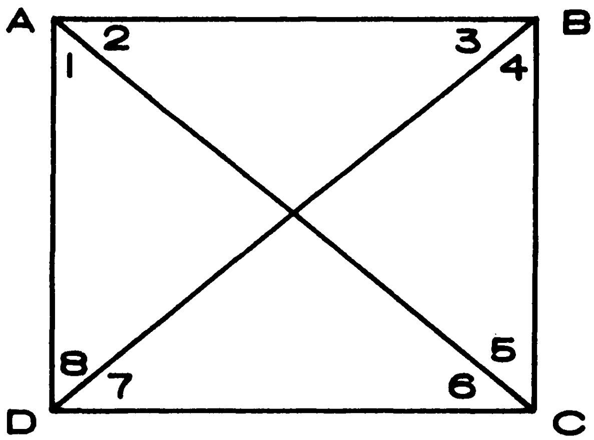
The 3 March calibration (Table 7), shows an approximate uncertainty in the control point coordinates of 0.16mm which is within the limits of accuracy expected from a calibration. The 22 April calibration (Table 8), which is a good quadrilateral as indicated by a mean error of $\pm$ 0.90" for the quadrilateral adjustment, breaks down to $\pm$ 0.66mm because of the coarse errors made when observing the control points. The $\pm$ 1.01mm uncertainty of the control point coordinates for the 6 May calibration, (Table 9), was due not only to careless quadrilateral observations (mean error for a single angle of $\pm$ 3.98"), but probably also to careless observations made on the control points.

Actually the coordinates of the six over-determined control points were taken as averages of the coordinates as obtained from each of the six baselines, and not by a least squares adjustment of the four directions. Little difference existed between the two methods, and either one produced

results comparable in reliability to the coordinates of the other 19 control points (see Table 10 which is a tabulation of results obtained by the two methods). Thus the difference between the two methods for the 3 March calibration is negligible. The quadrilateral for the 22 April calibration is good, but coarse errors were made when observing the control points. The observations for the 6 May quadrilateral are affected by coarse errors leading to wrong control point coordinates.

Part A: Quadrilateral Adjustment (3 March Quadrilateral)

(a) The following figure shows the angles of the quadrilateral as observed in the field.



	Observed Angles	v's from (e)	Adjusted Angles
(1)	42° 14' 47.4"	-0.84"	42° 14' 46.56"
(2)	47 32 06.2	-0.35	47 32 05.85
(3)	47 39 41.7	-0.79	47 39 40.91
(4)	42 26 57.8	-0.57	42 26 57.23
(5)	42 21 17.1	-1.09	42 21 16.01
(6)	47 25 05.3	-0.60	47 25 04.70
(7)	47 46 43.1	-1.04	47 46 42.06
(8)	42 33 27.0	-0.32	42 33 26.68

NOTE: To satisfy the condition that the sum of the angles of a triangle must equal 180°, v_4 and v_8 were changed by one unit in the second decimal place. This difference was due to rounding.

(b) For the adjustment of the quadrilateral, 3 angle equations and 1 side equation must be satisfied.

Angle Equations

Denoting the corrections for $\measuredangle 1, \measuredangle 2 .. \measuredangle 8$ by $v_1, v_2 .... v_8$, the 3 angle conditioned equations for

$$\Delta ABC: \quad (2) + (3) + (4) + (5) = 180^{\circ}\ 00'\ 02''.8$$
$$\Delta DAB: \quad (8) + (1) + (2) + (3) = 180\ \ 00\ \ 02.3$$
$$\Delta DCB: \quad (7) + (4) + (5) + (6) = 180\ \ 00\ \ 03.3$$

are

$$\begin{aligned} 1.\quad & v_2 + v_3 + v_4 + v_5 = -2''.8 \\ 2.\quad & v_8 + v_1 + v_2 + v_3 = -2''.3 \\ 3.\quad & v_7 + v_4 + v_5 + v_6 = -3''.3 \end{aligned} \qquad (1)$$

Side Equation

The logarithmic form of the side equation in terms of the corrections is:

$$-d_1v_1 + d_2v_2 - .... + d_8v_8 - \log\sin O_1 + \log\sin O_2 - .... + \log\sin O_8 = 0$$

or

$$- d_1v_1 + d_2v_2 - .... + d_8v_8 = \Theta$$

where, O_1, O_2 etc. are the observed values of the angles, d_1, d_2 etc. are the tabular differences for $1''$ for the log sines of O_1, O_2, etc. and Θ is the amount by which the observed values fail to satisfy the side equation.

Taking the unit as the 6th place of logarithms and arranging in tabular form,

Angle	+ log sin	d
8	9.83015851	2.29
6	9.86706149	1.93
4	9.82926432	2.30
2	9.86787422	1.93

Σ 39.39435854
39.394362265

4.11 = θ

Angle	- log sin	d
1	9.82757706	2.32
7	9.86955680	1.91
5	9.82847878	2.31
3	9.86875001	1.92

Σ 39.39436265

and the conditional side equation is

$$-2.32v_1 + 1.93v_2 - 1.92v_3 + 2.30v_4 - 2.31v_5 + 1.93v_6 - 1.91v_7 + 2.29v_8 = 4.11 \quad (2)$$

(c) Correlate Equations

Tabulating the conditional equations (1) and (2) in vertical columns,

v	k_1	k_2	k_3	k_4
1		1		-2.32
2	1	1		1.93
3	1	1		-1.92
4	1		1	2.30
5	1		1	-2.31
6			1	1.93
7			1	-1.91
8		1		2.29

(d) and forming the normal equations in the usual manner

	k_1	k_2	k_3	k_4		Absolute Column
1	4	2	2	0	=	-2.8
2	2	4	0	-.02	=	-2.3
3	2	0	4	+.01	=	-3.3
4	0	-.02	+.01	+36.0369	=	+4.11

whereby the roots of the normal equations are

$$k_1 = .11396002$$
$$k_2 = -.82514245$$
$$k_3 = -.57458775$$
$$k_4 = -.00028490$$

(e) Substituting these k's back into (c) the correlate equations, the v's are

$$v_1 = -.839$$
$$v_2 = -.354$$
$$v_3 = -.793$$
$$v_4 = -.563$$
$$v_5 = -1.088$$
$$v_6 = -.605$$
$$v_7 = -1.042$$
$$v_8 = -.313$$

where:

$$m.e. = \pm\sqrt{\frac{\sum[vv]}{n-u}} = \pm 1.06''$$

n = number measurements = 8

u = number of unknowns required for unique solution = 4

which when added to the observed angles result in the final adjusted values (see (a)).

COMPUTATION OF QUADRILATERAL ABCD

Using adjusted angles of quadrilateral ABCD and assuming

Coordinates of Sta. D,	X	= 100.0000 m
	Y	= 100.0000 m
Azimuth D ⟶ C		= $270^\circ\ 00'\ 00.00''$
Length CD (measured)		= 6.5977 m

the lengths and azimuths between quadrilateral stations ABC and D and the XY coordinates of the four stations are

Station	Length (Meters)	Azimuth °	'	"
AD	7.22566	359	39	51.26
BA	6.61130	89	52	58.85
CB	7.23910	179	46	20.71
DC	6.59770*	270	00	00.00**
AC	9.81318	317	25	04.70
DB	9.77521	222	13	17.94

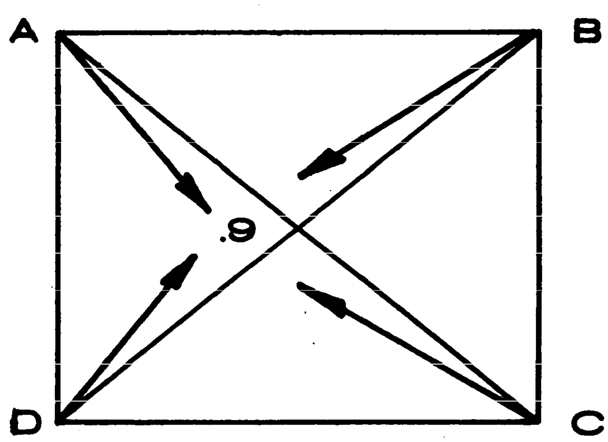
Station	X (Meters)	Y (Meters)
A	92.7745	100.0423
B	92.7610	93.4310
C	100.0000	93.4023
D	100.0000**	100.0000**

* measured

** assumed

Part B: Adjustment of the four directions to a control point.

As an example control point No. 9 of 3rd March quadrilateral will be adjusted.



Because of the errors in directions A-9, B-9, C-9 and D-9, these 4 lines will not intersect at a common point and must therefore be adjusted. An adjustment will be made so that the sum of the squares of the corrections to these 4 directions will be a minimum. The corrections contributed by these 4 corrected directions to the XY coordinates of the control point will also be computed.

The formulas used in this adjustment are similar to the ones developed in Reference (3).

Let

x, y = true coordinates

$(x), (y)$ = approximate coordinates (any reasonable value may be chosen; however, the better the approximations are the less will be the number of iterations required. The iterations should be repeated until the v's become stabilized)

$\Delta x, \Delta y$ = corrections to the assumed coordinates

then

$$x = (x) + \Delta x$$
$$y = (y) + \Delta y$$

Let

ϕ = adjusted direction

(ϕ) = approximate direction

α = measured direction

v = correction to measured direction

then

$$\phi = \alpha + v$$

Using the classical formula

$$\phi = (\phi) \frac{-\sin(\phi)\rho}{S^o} \Delta X + \frac{\cos(\phi)\rho}{S^o} \Delta Y$$

where S^o = distance from the station to the control point before adjustment,

and letting

$$a = \frac{-\sin(\phi)}{S^o}\rho \qquad \text{and} \qquad b = \frac{\cos(\phi)}{S^o}\rho$$

then

$$\phi = (\phi) + a\Delta X + b\Delta Y$$

$$\alpha + v = (\phi) + a\Delta X + b\Delta Y$$

Let

$$\ell = \alpha - (\phi)$$

then

$$v = a\Delta X + b\Delta Y - \ell,$$

This expression is the observation equation for a direction.

ADJUSTMENT OF CONTROL POINT NO. 9 (3 MARCH QUADRILATERAL)

Station A

Observed Angle				Adjusted AZ or Back AZ					Azimuth		
BA9	63°	43'	29"	AB	269°	52'	58".85		A-9	333° 36'	27".85
CA9	16	11	23	AC	317	25	04.70		A-9	333 36	27.70
DA9	26	03	25	AD	359	39	51.26		A-9	333 36	26.26
								Mean	A-9	333° 36'	27".27

Station B

AB9	36°	47'	15"	BA	89°	52'	58".85		B-9	53° 05'	43".85
CB9	53	19	25	BC	359	46	20.71		B-9	53 05	45.71
DB9	10	52	27	BD	42	13	17.94		B-9	53 05	44.94
								Mean	B-9	53° 05'	44".83

Station C

BC9	53°	02'	51"	CB	179°	46'	20".71		C-9	126° 43'	29".71
AC9	-10	41	34	CA	137	25	04.70		C-9	126 43	30.70
DC9	36	43	31	CD	90	00	00.00		C-9	126 43	31.00
								Mean	C-9	126° 43'	30".47

Station D

AD9	26°	06'	59"	DA	179°	39'	51".26		D-9	205° 46'	50".26
BD9	16	26	28	DB	222	13	17.94		D-9	205 46	49.94
CD9	64	13	11	DC	270	00	00.00		D-9	205 46	49.00
								Mean	D-9	205° 46'	49".73

$(x_a) = 96.38152$ m

$(y_a) = 98.25239$ m

are approximate coordinates, computed from baseline AB

A-9

$$\frac{\Delta y}{\Delta x} = \frac{1.789950}{3.60706} = .49623516$$

$$(\phi) = 360^\circ - \tan^{-1}\frac{\Delta y}{\Delta x} = 333^\circ\ 36'\ 27''.99 \qquad \cos(\phi)\ .89577209 \quad \sin(\phi)\ -.44451362$$

$s = 4026.76$mm

$\alpha = 333^\circ\ 36'\ 27''.27$

$v_A = 22.77\ \Delta x + 45.88\ \Delta y + 0.72$

B-9

$$\frac{\Delta y}{\Delta x} = \frac{4.821336}{3.620559} = 1.33165514$$

$$(\phi) = \tan^{-1}\frac{\Delta y}{\Delta x} = 53^\circ\ 05'\ 43''.65 \qquad \cos(\phi)\ .60048361 \quad \sin(\phi)\ .79963706$$

$s = 6029.41$mm

$\alpha = 53^\circ\ 05'\ 44''.83$

$v_B = -27.36\ \Delta x + 20.54\ \Delta y - 1.18$

C-9

$$\frac{\Delta y}{\Delta x} = \frac{4.850090}{3.618480} = 1.34036667$$

$$(\phi) = 180^\circ - \tan^{-1}\frac{\Delta y}{\Delta x} = 126^\circ\ 43'\ 31''.13 \qquad \cos(\phi)\ -.59797932 \quad \sin(\phi)\ .80151153$$

$s = 6051.18$mm

$\alpha = 126^\circ\ 43'\ 30''.47$

$v_c = -27.32\ \Delta x - 20.38\ \Delta y + 0.66$

D-9

$$\frac{\Delta y}{\Delta x} = \frac{1.747610}{3.61848} = .48296799$$

$$(\phi) = 180^\circ - \tan^{-1}\frac{\Delta y}{\Delta x} = 205^\circ\ 46'\ 44''.60 \qquad \cos(\phi)\ -.90047781 \quad \sin(\phi)\ -.43490196$$

$s = 4018.40$mm

$\alpha = 205^\circ\ 46'\ 49''.73$

$v_D = 22.32\ \Delta x - 46.22\ \Delta y - 5.13$

Observation Equations

$$22.77\,\Delta x + 45.88\,\Delta y + 0.72 = v_A = -0.6187$$
$$-27.36\,\Delta x + 20.54\,\Delta y - 1.18 = v_B = -3.0385$$
$$-27.32\,\Delta x - 20.38\,\Delta y + 0.66 = v_c = 0.6778$$
$$22.32\,\Delta x - 46.22\,\Delta y - 5.13 = v_D = -2.2638$$
$$\sum [vv] = 15.1995$$

Normal Equations

Δx	Δy	$-\ell$	
2511.61	7.86	- 83.8536	
	5078.50	232.4542	
	(- .02)	(.2624)	
		28.6633	
		(- 2.7996)	
	5078.48	232.7166	Δy = -.04582mm
		25.8637	
		- 10.6640	
		(15.1997) =	$\sum [vv]$

Δy	Δx	$-\ell$	
5078.50	7.86	232.4542	
	2511.61	- 83.8536	
	(- .01)	(- .3598)	
		28.6633	
		(- 10.6399)	
	2511.60	- 84.2134	Δx = +.03353mm
		18.0234	
		2.8236	
		(15.1998) =	$\sum [vv]$

$$\text{m.e.} = \pm\sqrt{\frac{\Sigma[vv]}{n-u}} = \pm 2\overset{''}{.}76$$

$$\text{m.e. } \Delta x = \frac{\pm 2.76}{\sqrt{2511.60}} = \pm .0551\text{mm}; \quad X = (x) + \Delta x = 96.38155 \text{ m}$$

$$\text{m.e. } \Delta y = \frac{\pm 2.76}{\sqrt{5078.48}} = \pm .0387\text{mm}; \quad Y = (y) + \Delta y = 98.25234 \text{ m}$$

A complete tabulation of results is given in Tables 7 to 10.

<u>Summary</u>

The frequency of pit calibrations depends on whether or not the relative changes in the positions of the control points are large enough to affect the camera orientations. However, the care with which a pit calibration is made is the controlling factor. Table 11 shows the results of RC-7 camera orientations using coordinates of a recent pit calibration (6 May) as opposed to a pit calibration made several months earlier (3 March). It is noted that the smaller mean errors for the RC-7 camera orientation result when using the coordinates obtained from the older calibration ($4.9 \angle 8.7$ and $5.9 \angle 8.4$). The residual errors for each control point are plotted in Fig. 6 for the 7 May orientations. The solid and broken arrows indicate the errors in the plate measurements resulting from the errors in the 3 March and 6 May pit calibrations.

Results of numerous pit calibrations indicated that the positions of the 25 control points could be determined within 0.2mm, with a somewhat better value for the two points used to establish the gun center-line. Quadrilateral observations with mean errors of a single angle less than 2 seconds of arc and control point observations with mean errors of a single direction less than 5 seconds of arc will assure this 0.2mm accuracy. A normal amount of care while making the field observations will produce results within these limits. It is important that both the quadrilateral and control point observations are within these limits since a relaxation of either one directly affects the final control point coordinates.

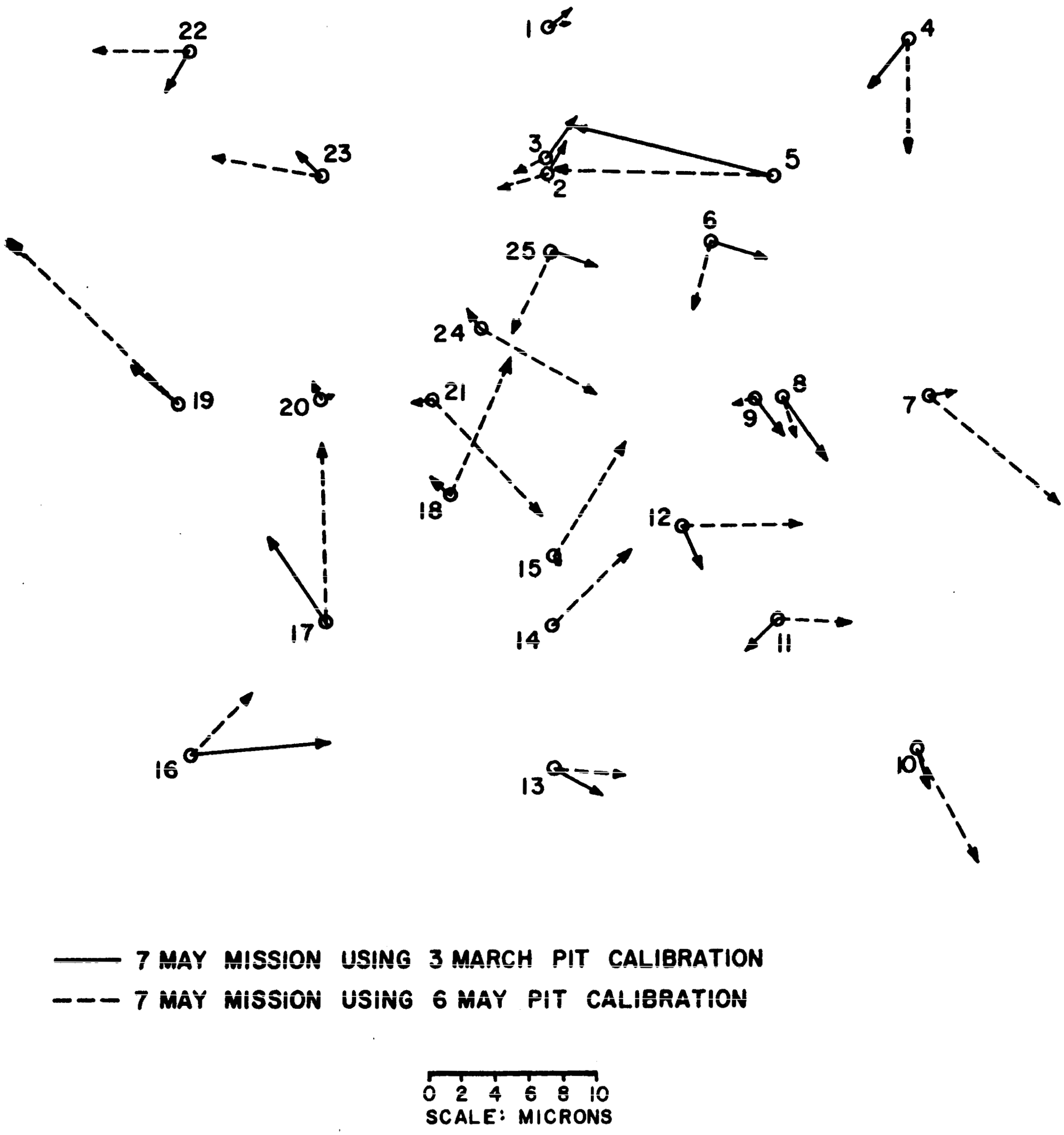


FIG. 6-RESIDUAL ERRORS OF RC-7 ORIENTATIONS OVER CALIBRATION PIT

3. RC-7 CAMERA
(Airborne Photogrammetric Camera)

3.1 Description and Function

The Wild RC-7 aerial camera (Fig. 7), an automatic plate changing type camera designed principally for mapping purposes, was used to determine the direction in which the gun was pointing at the moment it was fired. Two interchangeable types of cones were available, a 60° cone (Aviotar f/4.2 to f/16) with a focal length of 170mm, and a 90° cone (Aviogon f/5.6 to f/16) with a focal length of 100mm. The plate size is 15 x 15cm with a field of 14 x 14cm. All missions were run using the wide angle Aviogon at f/5.6 and with a shutter speed of approximately 1/100 second. The emulsion type used was Eastman Kodak spectroscopic 103F. A specially designed lens was temporarily attached to the RC-7 camera to accommodate the short distances encountered when photographing the control points in the calibration pit. Distortion corrections for this lens were taken into account for the RC-7 camera pit orientations.

During the calibration, when the aircraft was positioned over the calibration pit, a gun rod was temporarily inserted into the gun barrel and its direction was then determined by theodolite observations on the two control points mounted on the axis of the rod. During this time the RC-7 camera orientation was determined by photographing the 25 control points in the calibration pit (Fig. 4). The relation between the direction of the gun and the orientation of RC-7 camera obtained in the calibration, is assumed to remain constant during a mission. The direction of the gun at the moment of firing can then be determined from the orientation of the camera obtained from the photographed range control points (Fig. 8).

3.2 Comparator Code

All RC-7 plate measurements were corrected by a code, referred to as Comparator code, which was programmed for the ORDVAC computer. Included in this code are means to

(a) correct any systematic comparator errors

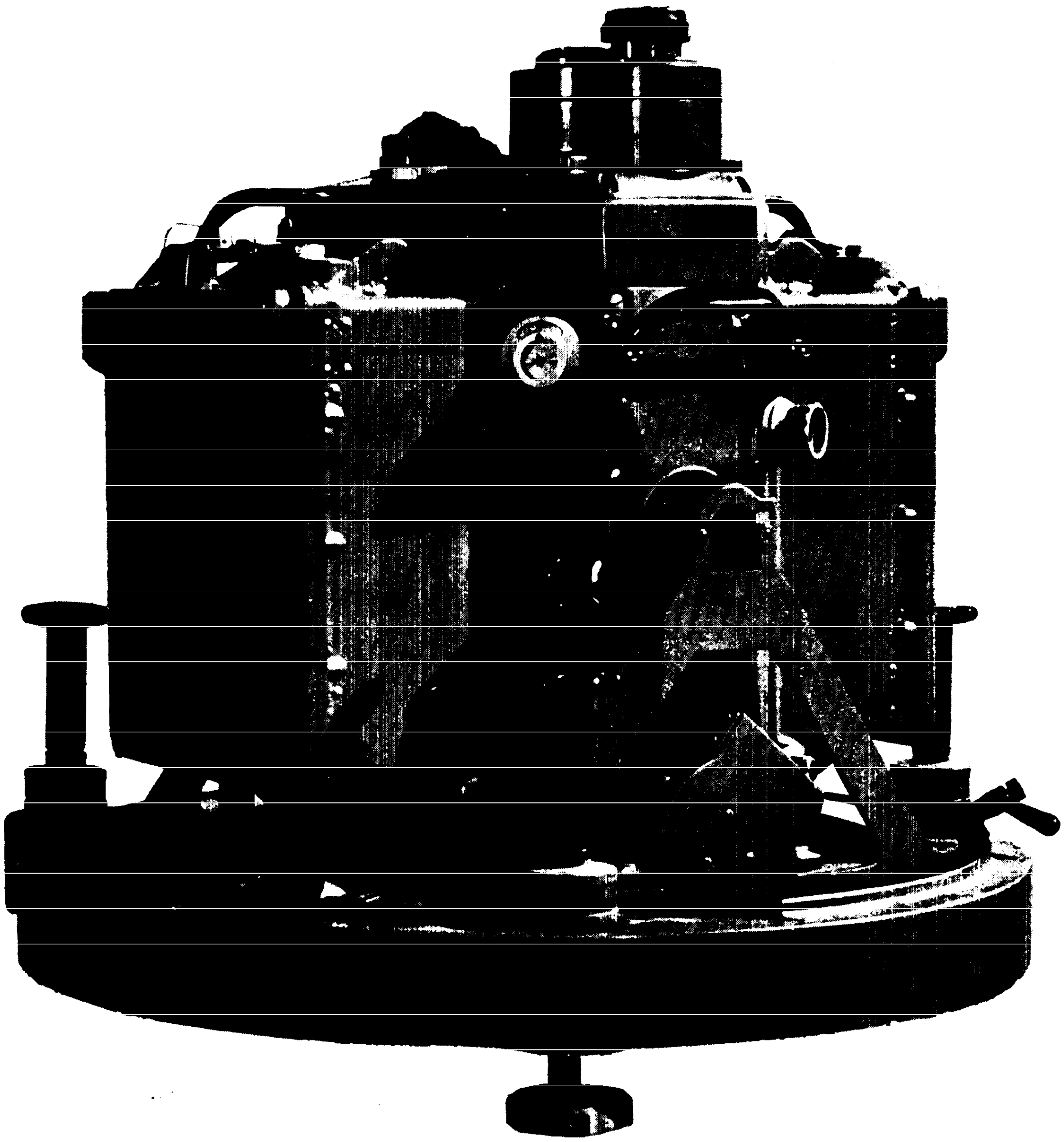


Fig. 7 RC-7 Aerial Camera

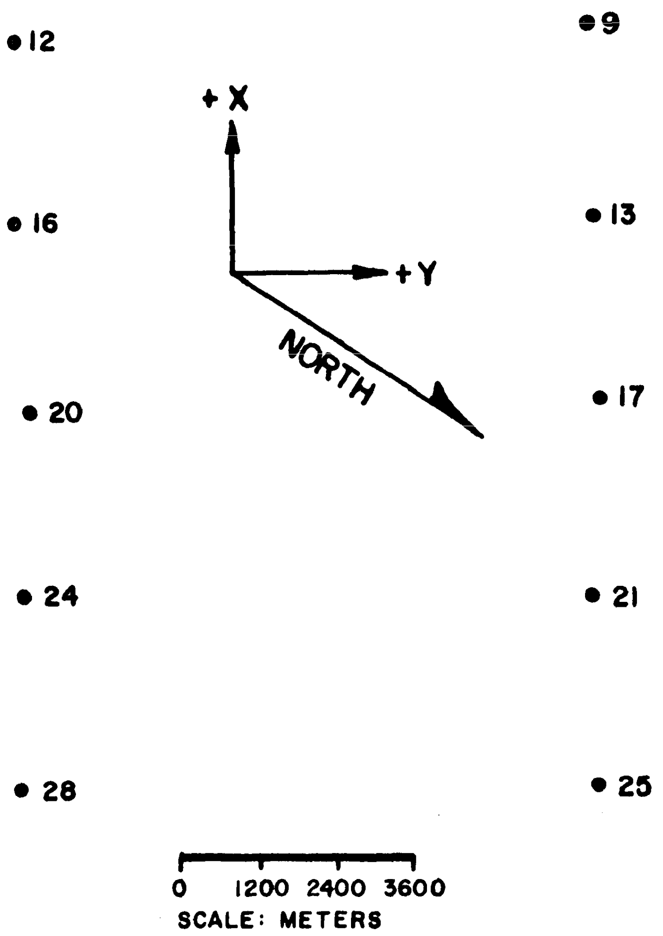


FIG. 8-RANGE CONTROL POINTS

(b) form arithmetic averages for multiple settings and if desired compute weight factors from the random discrepancies present in each set of measurements.

(c) reduce all plate measurements to an origin as defined by the intersection of two lines joining the two pairs of opposite fiducial marks.

(d) rotate the comparator system until it is parallel to any chosen set of fiducial marks.

(e) shift, if x_p and y_p are known, the origin to the principal point.

(f) apply lens distortion corrections.

The distortion curve used for RC-7 reductions over the range is

$$\Delta = + .00020072353r - .022916862r^3 - 17.634933r^5 + 2320.3473r^7$$

Distortion is positive from the center of the plate outward.

Table 12 gives a tabulation of the comparator code input and output values for the RC-7 camera over the range for 21 March M2/P1.

3.3 RC-7 Camera Orientation Reductions

RC-7 camera orientation reductions were made using equal weights for both the plate measurements and the control points. Equal weighting is justified if we assume the mean error of a single plate measurement to be $\pm$ 5.0 microns and the positions of the control points in the calibration pit and range known to $\pm$ 0.25mm and $\pm$ 50.0cm respectively. The following tabulation (Tables 13 and 14) gives the output from the camera orientation codes of the RC-7 camera over the calibration pit and the Range. These results show the residuals of the plate measurements and control points as obtained by equal weighting.

3.3.1 RC-7 Camera Orientation Elements

The analytical treatment of the orientation of a photogrammetric camera which has been applied in the present report is given in detail in an earlier report.[4]

The orientation parameters α, ω, κ , obtained from the RC-7 camera records in flights over the Range, are required to determine the gun direction. These orientation elements may be determined by enforcing and computing different combinations of the remaining six elements of orientation. It is necessary, if vertical photography is being reduced, to enforce Z_o, if c is to be computed. If x_p and y_p are to be computed, X_o and Y_o, respectively, must be enforced.

RC-7 camera orientations over the range were first determined by enforcing c, x_p, and y_p and computing α, ω, κ , X_o, Y_o and Z_o (for results of these orientations see Table 15). Later, when the Askania reductions were made, a systematic discrepancy was noted between the position of the RC-7 camera as determined by the above described method, and the position of the RC-7 camera as determined by the two ground-based Askania cameras, which triangulate the RC-7 position using stars as control points. A discussion of this discrepancy is given in section 3.4 of this report. Because the three elements of interior orientation of the Askania cameras are determined simultaneously with the triangulation results, it was decided that the RC-7 camera orientations should be determined by enforcing the three coordinates X_o Y_o Z_o as triangulated by the two Askania cameras. Thus the remaining six elements c, x_p, y_p, α, ω and κ were computed (Table 16).

Little difference exists between the final values of α, ω, κ , in Tables 15 and 16, regardless of what orientation elements are enforced or computed and either set of results could be used to satisfy the requirements for the gun direction. Table 17 is a tabulation of RC-7 orientation results over the calibration pit.

3.3.2 The orientation elements of the RC-7 camera comprise the elements of interior and of exterior orientation.

(a) The three elements of interior orientation are

c, the principal distance of the RC-7 camera, taken as the value given by the manufacturer.

x_p and y_p, the coordinates of the principal point, taken as the intersection of the two lines between opposite pairs of fiducial marks, consequently $x_p = 0$ and $y_p = 0$.

The elements of interior orientation should be determined by making an RC-7 cone calibration, taking star photographs. Such calibrations were not made for this project. Possible errors thus introduced into the elements of exterior orientation are discussed later in this report.

(b) The six elements of exterior orientation are

α, ω, κ, the three rotational elements which must be computed.

X_o, Y_o, Z_o, the coordinates of the camera center of projection.

3.4 RC-7 and Askania Comparisons

The three orientation elements (X_o Y_o Z_o) for the RC-7 camera can be determined by two independent methods

(1) Method of resection, by use of the RC-7 camera[4,5]

(2) Method of triangulation, by use of the two Askania phototheodolites[6]

The two methods should produce compatible results. Table 18 is a tabulation of results from several missions showing differences obtained by the two methods. The two values tabulated in columns (1) and (2) are the differences in the X_o and Y_o coordinates. The four values tabulated in columns (3), (4), (5) and (6) are differences in height. Column (3) shows the difference between the two methods before refraction corrections are made, column (4) shows the refraction correction, to be applied to the ground-based Askania cameras, column (5) shows the average refraction correction, to be applied to the airborne RC-7 camera, and column (6) shows the remaining difference in height as obtained by the two methods.

These differences could be attributed to:

(1) Errors in the absolute positions of the two Askania cameras.

(2) Errors in the absolute positions of the range targets, which were used to establish the RC-7 camera orientations.

(3) Errors in the plate measurements. Check runs (Table 18), were made by remeasuring the Askania and RC-7 camera plates and computing the XYZ coordinates from this new set of plate measurements. The differences between the two sets of results, which are due to errors in the plate measurements, indicate the reproducibility of the Askania and RC-7 comparisons. These results indicate that the errors in measuring the Askania and RC-7 camera plates produce differences in the X and Y coordinates of approximately two meters and differences in the Z coordinates of approximately one meter, corresponding to residuals in terms of the plate measurements of seven microns.

Table 15 is a tabulation of RC-7 camera orientation results for runs taken over the Range. These results were obtained by enforcing elements c, x_p, y_p and computing elements α, ω, κ, X_o, Y_o, Z_o. The average mean error of the plate measurements for the RC-7 camera over the Range is approximately $\pm$ 2.1 microns. The mean errors for the X_o Y_o and Z_o orientation elements are seen to be well within the 2.0 and 1.0 m differences that were obtained by the check runs.

(4) Errors in the interior elements of orientation of the RC-7 camera Reference (5). These are to be expected because the RC-7 camera was not calibrated.

(a) Errors in the values of x_p and y_p directly affect X_o and Y_o. An average error in X_o and Y_o of approximately 5.0 m (Table 18), would be explained by an error in x_p and y_p of 50 microns.

The actual differences between ΔX and ΔY values, as can be seen from Table 18, have a range of approximately 12 m. This range of values is probably due to different positions occupied by the RC-7 plates when

pressed onto the frame of the lens cone. This lack of exact positioning seems to be demonstrated by missions flown on the 10th and 11th of April (Table 18), when two consecutive photographs were taken during one pass. Disregarding the errors involved in the plate measurements, the differences between the ΔX's and ΔY's, regardless of their numerical values should agree, because only another RC-7 plate was introduced into the system. The fact that they disagree by amounts up to 5.0 m can only be explained by the failure of the plates to occupy identical positions when pressed onto the frame of the lens cone.

(b) Errors in the focal length of the RC-7 camera directly affect Z_o. Final differences between the two methods are tabulated in column (6) of Table 18. These differences are nearly constant, amounting to approximately three meters.

3.5 Refraction

Refraction corrections were made using formulas based on the assumption of a flat earth and NACA standard-atmosphere data. The formulas used are[7]

$$R.\,E. = \alpha_o \cot E_o \left\{ 1 + \frac{RT_o}{Y} \left[\left(1 + \frac{L}{T_o} Y\right)^{-\frac{1}{LR}} - 1 \right] \right\}$$

$$R.\,E.' = \alpha_o \cot E_o \left\{ \frac{RT_o}{Y} - \left(1 + \frac{L}{T_o} Y\right)^{-\frac{1}{LR} - 1} \left[1 + \frac{RT_o}{Y} \left(1 + \frac{L}{T_o} Y\right) \right] \right\}$$

where

RE = refraction error in radians for the ground station

RE' = refraction error in radians for the air station

α_o = .0002762

E_o = observed elevation angle

R = 29.2745m/°C

T_o = 288.16° K

Y = vertical distance between ground and air stations in meters

L = - .0065° C/m

Refraction corrections should be applied simultaneously with the triangulation computations. The corresponding rigorous treatment is developed in another report.[8] The presented results differ insignificantly from the rigorous approach.

In the case of the Askania results the lack of intersection between corresponding rays includes refraction influences. These discrepancies were adjusted by a least squares solution as though they were caused only by measuring errors. The refraction corrections were then computed for each of the two Askanias; designated Pi C and Pi D, in columns (1) and (2), Table 19. The difference between the two heights (column (3)) indicates that a bias was introduced by this approximation method. A mean value (column (4)) was used as the correction to be applied to the triangulated heights.

In the case of the RC-7 camera, a refractive height correction was computed as an average value obtained for the different range targets from several missions.

3.6 Reduction of Gun Direction

The direction of the gun axis at the moment the gun is fired was determined by the following method, which was coded for the ORDVAC*.

(a) The orientation of the RC-7 camera over the calibration pit is determined using an arbitrary +X direction for the plate coordinates system (3.2(d)). The same +X direction must be chosen for RC-7 plates taken over the range. This is necessary since any difference with respect to the orientation of the plate coordinate system during the process of measuring the plates would lead to a difference in the orientation element κ , and influence the gun direction.

---

*As input, the coordinate differences of the two control points on the gun rod and the elements of orientation from the RC-7 camera reductions, as obtained over the calibration pit and range, were used.

(b) From the results of the calibration pit survey, the differences in the XYZ coordinates of the two control points simulating the gun axis are computed (Fig. 9) where O and R are the upper and lower control points respectively.

$$(X)_R = (X_R - X_o)$$
$$(Y)_R = (Y_R - Y_o) \qquad (1)$$
$$(Z)_R = (Z_R - Z_o)$$

(c) Shifting the direction of the gun parallel to itself, into the center of projection of the RC-7 camera as obtained over the calibration pit, a set of pseudo plate coordinates x_g and y_g are computed by means of formula (12) of Reference (8). (2)

(d) The orientation of the RC-7 camera over the range is determined using the same X direction for the plate coordinate system as described in 3.6(a).

(e) Using the set of pseudo plate coordinates x_g and y_g from 3.6(c) and the RC-7 camera orientation over the range from 3.6(d) standard coordinates ζ and η are computed by means of formula (11) of Reference (8) and Fig. 9, this report. (3)

(f) From the computed values of ζ and η the direction of the gun over the range is determined by the following formulas:

$$\tan A = \frac{\eta}{\zeta} \qquad (4)$$

$$\tan \nu = \frac{\sqrt{\zeta^2 + \eta^2}}{r} \qquad (5)$$

or

$$\tan \alpha = \frac{\zeta}{r} = \tan \nu \cos A \qquad (6)$$

$$\tan \omega = \frac{\eta}{(\zeta^2 + 1^2)^{1/2}} = \tan A \sin \alpha \qquad (7)$$

where r = +1 or -1, depending on geometry.

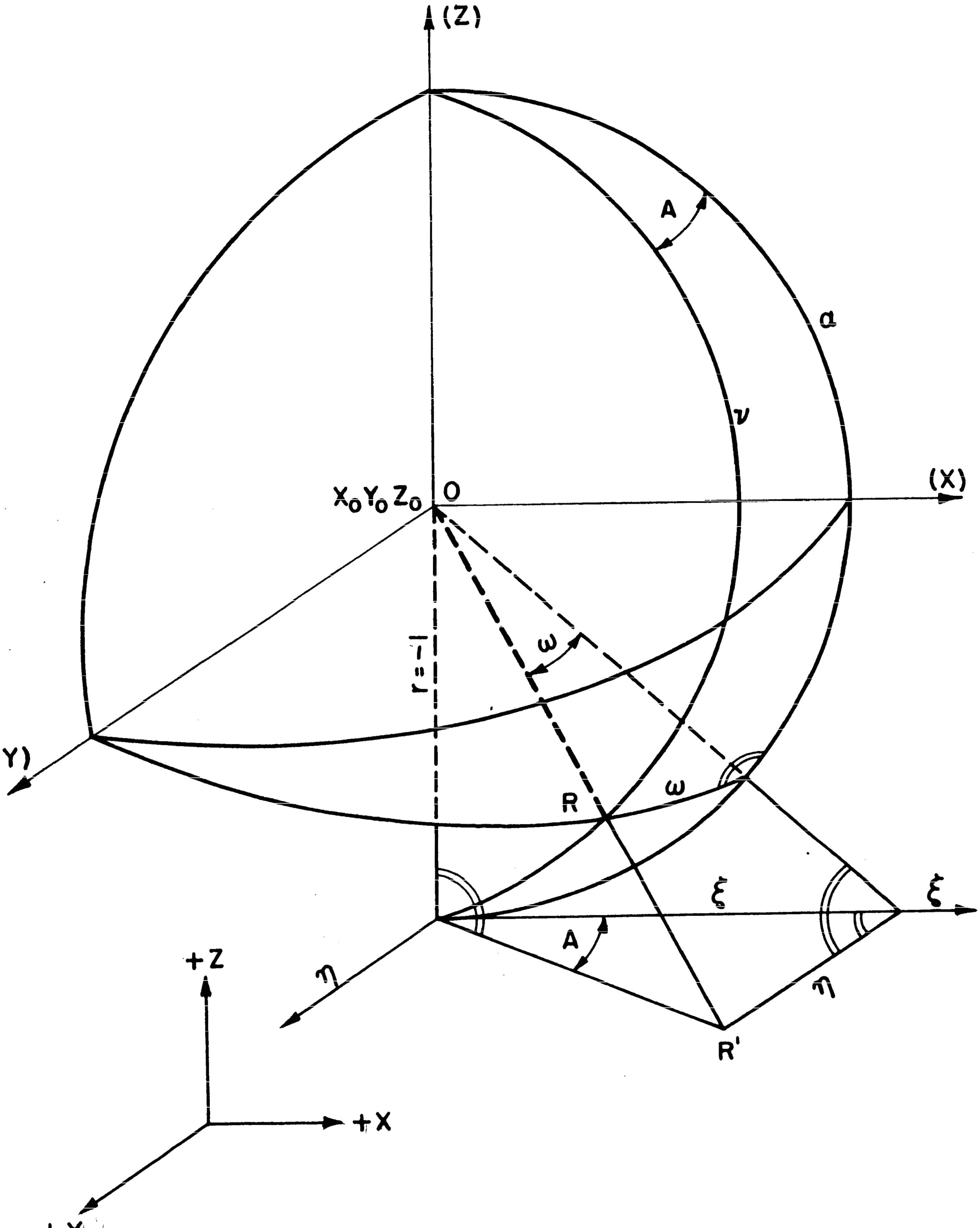


FIG. 9

3.7 An example of the computations, determining the gun direction for the 21 March M1/P2 firing follows:

(a) The orientation elements for the RC-7 camera over the calibration pit are (Table 17)

$$c = .10055 \text{ m}$$
$$x_p = 0 \text{ m}$$
$$y_p = 0 \text{ m}$$
$$\alpha = 198.051 \text{ grads}$$
$$\omega = - .045 \text{ grads}$$
$$\kappa = - 49.439 \text{ grads}$$

(b) Using formula (1) and referring to Table 6

$$(X)_R = + .0178 \text{ m}$$
$$(Y)_R = - .0007 \text{ m}$$
$$(Z)_R = - .8521 \text{ m}$$

(c) From formulas (12) and (13) of reference (8) the pseudo plate coordinates are

$$x_g = \frac{c\left[(X)_R A_1 + (Y)_R B_1 + (Z)_R C_1\right]}{q} + x_p = - .0007058$$

$$y_g = \frac{c\left[(X)_R A_2 + (Y)_R B_2 + (Z)_R C_2\right]}{q} + y_p = + .0006774$$

where $q = (X)_R D + (Y)_R E + (Z)_R F = + .85224589$,

and

$$A_1 = - \cos\alpha \cos\kappa + \sin\alpha \sin\omega \sin\kappa = + .7129913$$
$$B_1 = - \cos\omega \sin\kappa = + .7008482$$
$$C_1 = \sin\alpha \cos\kappa + \cos\alpha \sin\omega \sin\kappa = + .0213392$$
$$A_2 = - \cos\alpha \sin\kappa - \sin\alpha \sin\omega \cos\kappa = - .7005044$$
$$B_2 = \cos\omega \cos\kappa = + .7133103$$
$$C_2 = \sin\alpha \sin\kappa - \cos\alpha \sin\omega \cos\kappa = + .0219570$$
$$D = \sin\alpha \cos\omega = + .0306100$$
$$E = \sin\omega = - .0007069$$
$$F = \cos\alpha \cos\omega = - .9995312$$

where A, to F are essentially direction cosines.

(d) The orientation elements for the RC-7 camera over the range are (Table 16)

$$c = .100594 \text{ m}$$
$$x_p = - .000100 \text{ m}$$
$$y_p = + .000097 \text{ m}$$
$$\alpha = 197.819 \text{ grads}$$
$$\omega = - .159 \text{ grads}$$
$$\kappa = - 53.159 \text{ grads}$$

(e) From formulas (11) and (13) of reference (8) the standard coordinates of ζ and η corresponding to the range coordinate system are

$$\xi = \frac{r\left[(x_g - x_p)A_1 + (y_g - y_p)A_2 + cD\right]}{Q} = + .0259460$$

$$\eta = \frac{r\left[(x_g - x_p)B_1 + (y_g - y_p)B_2 + cE\right]}{Q} = - .0030904$$

where $Q = (x_g - x_p)\,C_1 + (y_g - y_p)C_2 + cF = -.10056317$ and $r = -1$, and from 3.7(c)

A_1	=	+ .6708327	A_2	=	- .7408173	D	=	+ .0342523
B_1	=	+.7413074	B_2	=	+.6711610	E	=	- .0024976
C_1	=	+ .0211385	C_2	=	- .0270670	F	=	- .9994101

(f) Using the computed values of ξ and η, the direction of the gun at the moment of firing, referred to the range coordinate system, is given by (3.6(f))

$$\tan A = - .1191089$$
$$A = + 392.453 \quad \text{grads}$$
$$\tan \nu = - .0261294$$
$$\nu = + 198.337 \quad \text{grads}$$

or

$$\tan\alpha = -\ .0259460$$

$$\alpha = +\ 198.349 \quad \text{grads}$$

$$\tan\omega = -\ .0030894$$

$$\omega = +\ 399.803 \quad \text{grads}$$

The results of the hand computed example above may differ slightly from the results obtained from the electronic computer because of the smaller number of digits carried. The results of all firings are shown in Table 20 and also in Table 27.

3.8 Accuracy of Gun Center Line at the Moment of Firing

Factors affecting the accuracy with which the gun direction at the moment of firing can be determined are

(a) maintaining relationship between gun barrel and RC-7 camera
(b) measuring direction of gun over calibration pit
(c) determining the direction of RC-7 over the calibration pit
(d) determining the direction of RC-7 over the range

(a) A post-calibration was attempted for the purpose of determining any relative differences between the gun center line and RC-7 camera before and after a mission. However, because of movements in the aircraft probably due to heat caused by its operation, pointings with the T-2 theodolites could not be made on the gun rod. If a check on this relationship is desired, the theodolites could be replaced by phototheodolites, for example the Wild Phototheodolite, which is a combination of a T-2 theodolite and camera carrier. Records could then be obtained by exposures taken simultaneously with the phototheodolites and the RC-7 camera.

(b) The error in the direction of the gun over the calibration pit is a result of errors in the X and Y coordinates of control points Nos. 26 and 27, which in turn are due to errors in the horizontal angles turned from stations A and D.

According to the method in 2.4(c) part B, a set of observational equations and corresponding normal equations can be established for the intersection of points Nos. 26 and 27.

The diagonal terms of the inverse of the resulting normal equation system, denoted by Q_x and Q_y give the weighting factors for the x and y coordinates, and their mean errors are obtained by

$$m_x = m \sqrt{Q_x}$$
$$m_y = m \sqrt{Q_y}$$

where m denotes the mean error of a single observation of unit weight

Applying this method to the intersection of point No. 26, we have

Coordinates	X	Y
Station A	92.8	100.0
Station D	100.0	100.0
Approx. values for Point No. 26	97.4	96.7

The observational equations are

(1) $21{,}250\, \Delta X + 29{,}600\, \Delta Y = \ell_1$

(2) $38{,}600\, \Delta X - 30{,}400\, \Delta Y = \ell_2$

and the inverse of the corresponding normal equations is

$$\begin{vmatrix} Q_x & Q_{xy} \\ Q_{xy} & Q_y \end{vmatrix} = \begin{vmatrix} .56279 \times 10^{-9} & .17021 \times 10^{-9} \\ .17021 \times 10^{-9} & .60693 \times 10^{-9} \end{vmatrix}$$

Consequently

$$m_x = \pm 0.047\text{mm}$$
$$m_y = \pm 0.049\text{mm}$$

where m was assumed to be $\pm 2''$ of arc.

Control point No. 27 is approximately below point No. 26; therefore, the mean errors for control points Nos. 26 and 27 will be the same. Combining the mean errors of control points Nos. 26 and 27, the mean error of the direction of the gun in the X direction is

$$\frac{(m_{X_{26}}^2 + m_{X_{27}}^2)^{1/2}}{d} \rho'' = \frac{.066}{852} \rho'' = 16''$$

and in the Y direction

$$\frac{(m_{Y_{26}}^2 + m_{Y_{27}}^2)^{1/2}}{d} \rho'' = \frac{.069}{852} \rho'' = 17''$$

where d = 0.852 meters, the distance between control points Nos. 26 and 27.

(c) The average error in the direction of the RC-7 camera over the calibration pit is (Table 17)

$\alpha = .008^g = 26''$, in the X direction
$\omega = .006^g = 19''$, in the Y direction

(d) The average error in the direction of the RC-7 camera over the range is (Table 16)

$\alpha = .005^g = 16''$, in the X direction
$\omega = .005^g = 16''$, in the Y direction

Combining the errors due to (b), (c) and (d) the error of the gun center line in the X and Y direction is expressed with sufficient accuracy by

$$\text{error } X_{dir} = \left[(16)^2 + (26)^2 + (16)^2\right]^{1/2} = 34''$$
$$\text{error } Y_{dir} = \left[(17)^2 + (19)^2 + (16)^2\right]^{1/2} = 30''$$

which is well within the limits set by the requirement of 1 mil (202.5").

4. ASKANIA CAMERAS
(Ground Based Photogrammetric Cameras)

4.1 Description and Function

The Askania camera (Fig. 10), a phototheodolite designed primarily for the determination of trajectories, was used to determine the following quantities:

(1) Space coordinates of the aircraft at the moment of firing

(2) Space coordinates of the burst of the projectile

(3) Velocity of the aircraft

Two phototheodolites, at stations Pi C and Pi D, were used.

The Askania camera is equipped with a 24° cone, relative aperture variable from f/5.5 to f/64, a focal length of approximately 370mm and a plate size of 13 x 18cm. All missions were run using an aperture of f/5.5 and Eastman Kodak 103F emulsion.

4.2 Coordinates of the Askania Cameras

The geographic positions of the two Askania cameras and the Range lights serving as control points for the RC-7 camera, were determined by survey ties from United States Coast and Geodetic Survey triangulation stations. When reduced to a rectangular coordinate system the Askania camera stations and the Range lights are compatible. However, because of the possible systematic orientation error in the U.S.C. and G.S. triangulation stations, the flashes as triangulated by the ground based Askania cameras and the results obtained by the RC-7 camera may not be compatible. A suggested solution is to use three ground based cameras following the method of reduction as outlined in reference (9).

4.3 Coordinate Transformation

An XYZ rectangular coordinate system, using Range station MID as the origin, was set up for the Range target lights and the two Askania cameras as follows[10]:

+ X, increasing in the direction of flight, at an angle of $56^{\circ}\ 56'\ 41''$ measured clockwise from the south

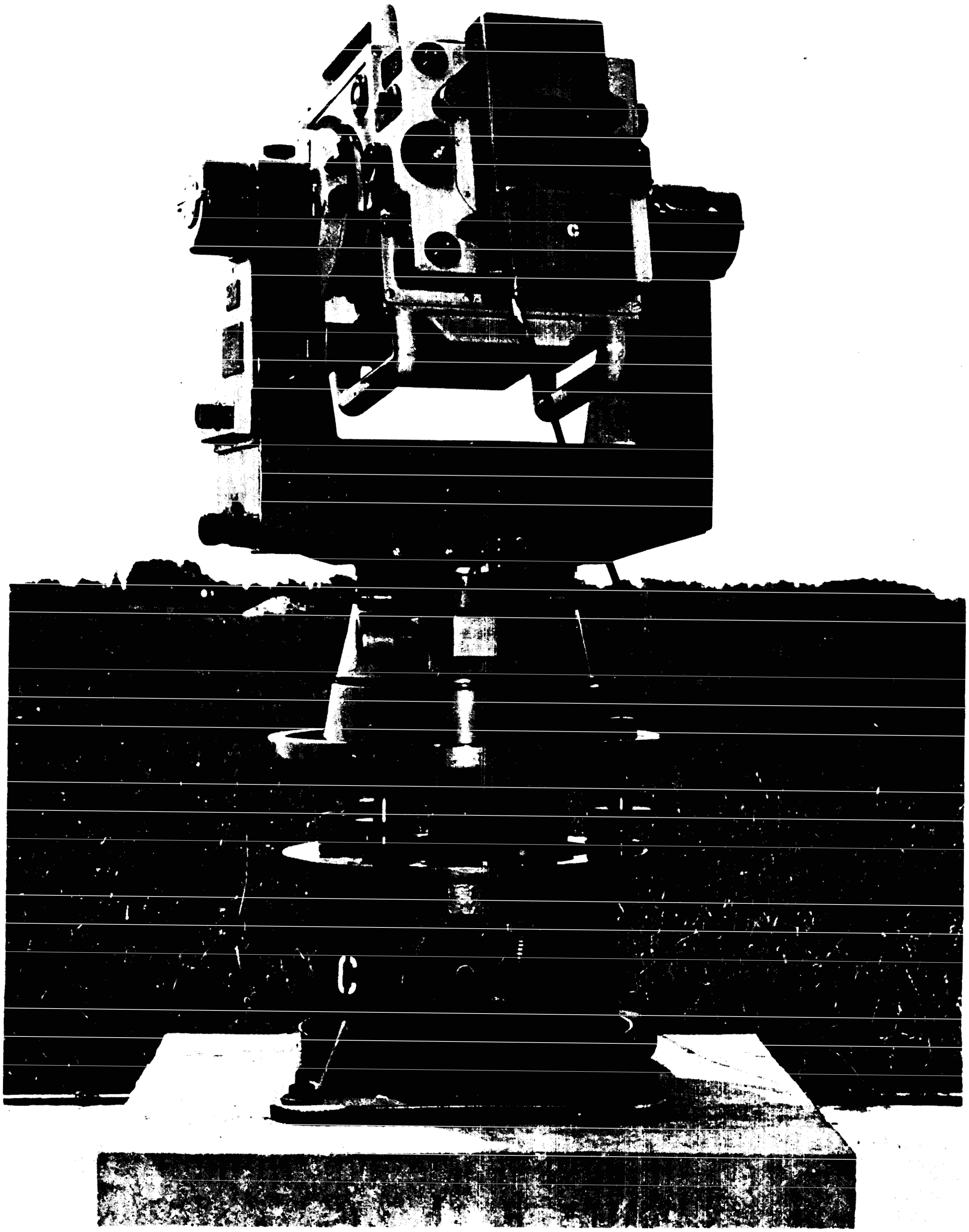


Fig. 10 Askania Phototheodolite

+ Y, 90° clockwise from + X

+ Z, measured positive upwards from a plane tangent to the reference ellipsoid at station MID

The values to be used in connection with Reference (10) are

$$
\begin{aligned}
H_o &= 0 \\
\phi_o &= 30^{\circ}\ 35'\ 01''.732 \\
\lambda_o &= (\lambda_8) = 86^{\circ}\ 39'\ 59''.275 \\
\alpha &= 59^{\circ}\ 24'\ 58''.268 \\
\beta &= 56^{\circ}\ 56'\ 41'' \\
\gamma &= 0 \\
a &= 6,378,206.4 \text{ m} \\
b &= 6,356,583.8 \text{ m}
\end{aligned}
$$

A tabulation of input (ϕ, λ, H) and output (XYZ) is given in Tables 21 and 22.

4.4 Askania Camera Orientations.[4]

The absolute orientations of the Askania cameras were obtained by using stars as control points (Fig. 11).

These control points are given in the form of Standard Coordinates (ξ η), which are obtained from their apparent positions (RA, δ) at the mean time of the observation by the equations on page 15, Reference (9):

$$
\begin{aligned}
+\ \xi \text{ (north)} &= -\tan z_r \cos A \\
+\ \eta \text{ (east)} &= -\tan z_r \sin A \\
+\ Z &= +\ 1.0 \text{ (unit)}
\end{aligned}
$$

where, from the astronomical triangle,

$$
\begin{aligned}
\cos z &= \sin \delta \sin \phi + \cos \delta \cos \phi \cos t \\
\sin z \cos A &= -\sin \delta \cos \phi + \cos \delta \sin \phi \cos t \\
\sin z \sin A &= \cos \delta \sin t
\end{aligned}
$$

z = zenith distance of the star

ϕ = geographic latitude of the camera station

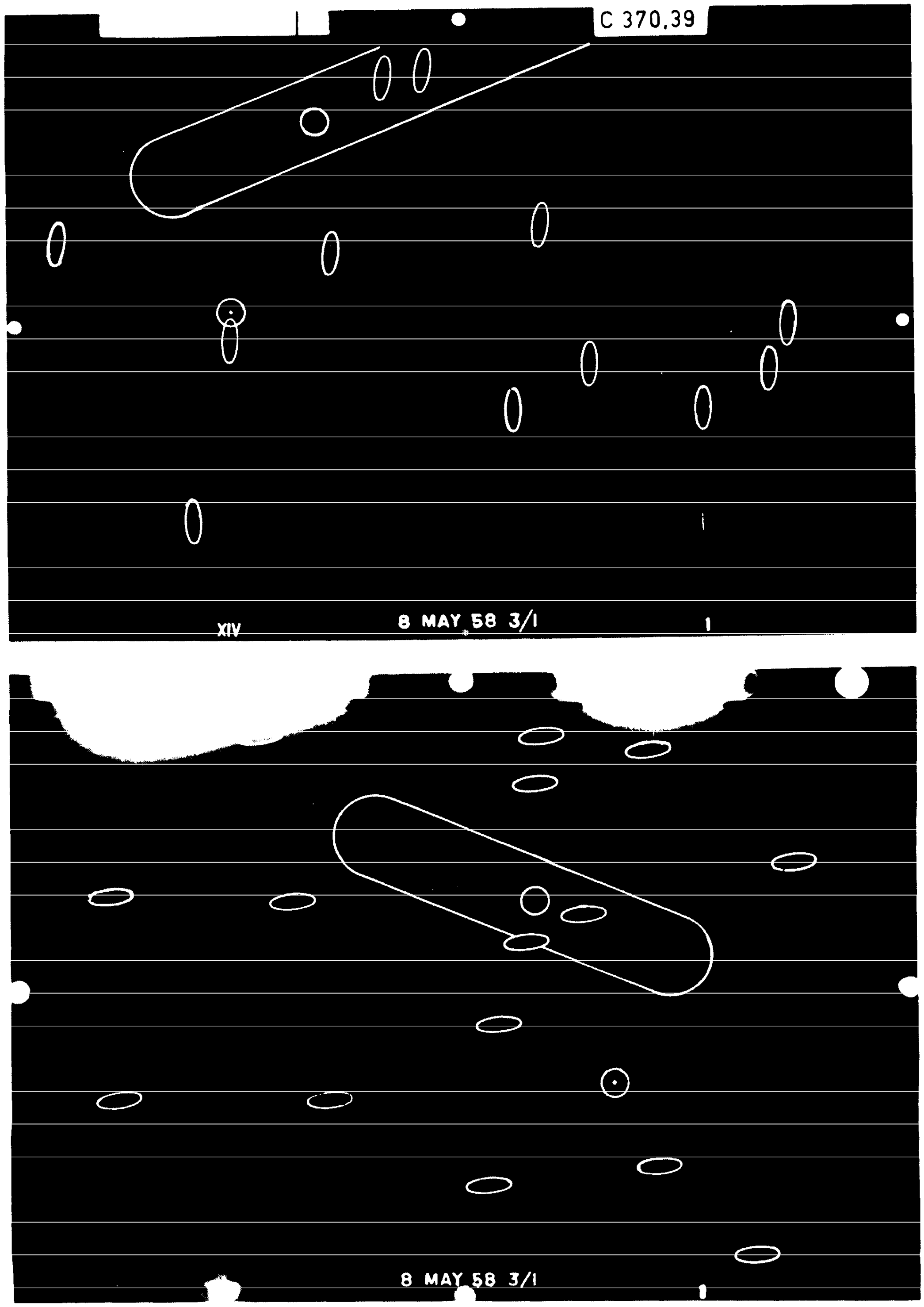


Fig. 11 Pi C and Pi D Askania Camera Exposures Taken on 8 May M3/P1

The stars are marked by oval shaped circles, the flashes emitted from the aircraft during the mission by the large oval shaped figure, and the projectile burst is marked by a small circle. The small circle inside the large oval shaped figure contains the RC-7 flash that occurs near the time at which the gun was fired.

$\delta =$ declination of the star

$t =$ hour angle of the star

$A =$ azimuth of the star, measured clockwise from south

and subscript r indicates that the zenith distance has been corrected for astronomical refraction.

The nine orientation elements of each Askania camera are similar to the ones described under the RC-7 camera orientation elements, the three coordinates of the Askania camera center of projection ($X_oY_oZ_o$) are the center of the unit sphere and the remaining six orientation elements (α, ω, κ, c, x_p, y_p) are computed.

The number of absolute control points (stars) required for a unique solution is three, however, in most cases from 8 to 15 stars were chosen. Tables 23 and 24 give tabulations of the number of stars used, the six computed orientation elements and the corresponding mean errors for the Askania camera orientation reductions.

4.5 Triangulation of Flashes and Burst

(a) The two Askania cameras were located at the ends of an 11-km baseline which was approximately perpendicular to the direction of flight of the aircraft. The cameras were pointed approximately 30 degrees down-range from each other and at elevation angles of approximately 56 degrees. As the aircraft flew between the two Askanias, a pre-established sequence of flashes was emitted consisting of 10 pre-survey flashes (Nos. 1 to 10), an RC-7 flash (No. 11), a reset flash (No. 12) and 10 post-survey flashes (Nos. 13 to 22). The 10 post-survey flashes served as the pre-survey flashes for the next round fired.

The flashes and burst were observed by photoelectric cells and recorded on the ground against a time base. In most cases flash No. 11 was the first flash to be picked up. Table 25 gives a tabulation of actual times referred to an arbitrary zero time.

Because of the extreme brightness of the burst, a large image appeared on the plate. In order to increase the accuracy of identifying the center of the burst a diffraction effect was produced by placing a fine mesh screen in front of the Askania lens. Accurate settings could then be made by using the resulting cross-shaped pattern (Fig. 11).

(b) A computed example of the triangulation of the RC-7 flash from Pi C and Pi D Askania cameras for 7 May M1/P1 follows.

The absolute orientation of each camera is obtained from stars photographed on the same plate with the flash points (Fig. 11). The direction of each individual flash point can therefore be determined and triangulation can proceed according to the methods outlined in Reference (6).

The length of the base line (about 11,000 m) necessitates the use of either a spherical or ellipsoidal rectangular system when triangulating the flashes since the absolute orientation of each camera refers to the local zenith. Both of these methods are somewhat longer than the simpler solution in the Cartesian system used later for the velocity cameras. It is, however, possible to apply a transformation to the constants A_1, B_1, C_1, A_2, B_2, C_2, D_1, E_1, F_1 which will enable the triangulation to be carried out in the Cartesian system.

In Appendix C Reference (11), it is pointed out that the nine constants derived from the orientation are actually the direction cosines of one orthogonal system with respect to another. More precisely, these are the direction cosines of the $l_x - x_p$, $l_y - y_p$, coordinate system with respect to an XYZ system in which the X-axis is positive toward the north, the Y-axis is positive toward the east and the Z-axis is positive in the direction away from the center of the earth. It is therefore possible to make a transformation of the coordinates of one camera to a system parallel with the system of the second camera.

A local Cartesian system was set up, see (4.3), in which the coordinates of the flash point are to be expressed. Hence, a transformation may be set up for each station by which the orientation elements of the two Askanias are reduced to the local Cartesian system. This transformation may be conveniently written by means of the method outlined in Sec. 6 of Reference (11).

The data for the Askania stations and the origin, which are used in the computation are as follows:

	Origin (Station MID)	Pi C	Pi D
ϕ	$30^\circ\ 35'\ 01''.732$ N	$30^\circ\ 33'\ 00''.502$ N	$30^\circ\ 37'\ 57''.288$ N
λ	86 39 59.275 W	86 37 19.432 W	86 41 02.675 W

Using the notation on p. 27 of Reference (11), we have the following 3 x 3 matrices:

$$\Psi = \begin{bmatrix} \cos\phi_o & 0 & -\sin\phi_o \\ 0 & 1 & 0 \\ \sin\phi_o & 0 & \cos\phi_o \end{bmatrix} = \begin{bmatrix} .86088579 & 0 & -.50879824 \\ 0 & 1 & 0 \\ .50879824 & 0 & .86088579 \end{bmatrix}$$

$$\Omega_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\lambda_i & \sin\Delta\lambda_i \\ 0 & -\sin\Delta\lambda_i & \cos\Delta\lambda_i \end{bmatrix} \qquad \text{where } \Delta\lambda_i = \lambda_o - \lambda_i$$

Pi C

$$\Delta\lambda = 0^\circ\ 02'\ 39''.843$$

Pi D

$$\Delta\lambda = -(0^\circ\ 01'\ 03''.400)$$

$$\Omega_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .99999970 & .00078214 \\ 0 & -.00078214 & .99999970 \end{bmatrix} \qquad \Omega_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .99999995 & -.00030737 \\ 0 & .00030737 & .99999995 \end{bmatrix}$$

$$\hat{\Psi}_i = \begin{bmatrix} \cos\phi_i & 0 & -\sin\phi_i \\ 0 & 1 & 0 \\ \sin\phi_i & 0 & \cos\phi_i \end{bmatrix}$$

$$\hat{\Psi}_i = \begin{array}{cc} \text{Pi C} & \text{Pi D} \\ \begin{bmatrix} .86118468 & 0 & -.50829218 \\ 0 & 1 & 0 \\ .50829218 & 0 & .86118468 \end{bmatrix} & \begin{bmatrix} .86045243 & 0 & -.50953077 \\ 0 & 1 & 0 \\ .50953077 & 0 & .86045243 \end{bmatrix} \end{array}$$

If $\Gamma_i = \Psi_o \ \Omega_i \ \hat{\Psi}_i^{\ T}$, we have:

$$\Gamma_i = \begin{array}{cc} \text{Pi C} & \text{Pi D} \\ \begin{bmatrix} .99999974 & .00039795 & -.00058761 \\ -.00039756 & .99999970 & .00067357 \\ .00058787 & -.00067333 & .99999960 \end{bmatrix} & \begin{bmatrix} .99999961 & -.00015639 & .00085114 \\ .00015661 & .99999995 & -.00026448 \\ -.00085110 & .00026461 & .99999959 \end{bmatrix} \end{array}$$

Let A be the transformation due to the rotation of the XY-plane in azimuth. Then

$$A = \begin{bmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -.54544791 & -.83814472 & 0 \\ .83814472 & -.54544791 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And finally the complete transformation $A\Gamma_i$ is:

$$\begin{array}{cc} \text{Pi C} & \text{Pi D} \\ \begin{bmatrix} -.54511456 & -.83836153 & -.00024404 \\ .83836135 & -.54511421 & -.00085990 \\ .00058787 & -.00067333 & .99999960 \end{bmatrix} & \begin{bmatrix} -.54517896 & -.83805938 & -.00024258 \\ .83805897 & -.54557896 & .00085764 \\ -.00085110 & .00026461 & .99999959 \end{bmatrix} \end{array}$$

The following values of α, ω, κ , c, x_p, y_p were derived from the orientations of the 2 cameras (Tables 23 and 24):

	Pi C					Pi D				
α =	24.373781 grad	21°	56'	11".05		-41.756238 grad	-37°	34'	50".21
ω =	-35.260531 "	-31	44	04.12		4.9600658 "	4	27	50.61
κ =	-57.607548 "	-51	50	48.46		194.39737 "	174	57	27.48
c =	.37050712 m					.37035596 m			
x_p =	.16663650 "					.07960646 "			
y_p =	.19518634 "					.09748358 "			

It is now convenient to write the constants A_1, B_1, C_1, A_2, B_2, C_2, D_1, E_1, F_1 in the form of a 3 x 3 matrix as follows:

$$\begin{bmatrix} A_1 & A_2 & D_1 \\ B_1 & B_2 & E_1 \\ C_1 & C_2 & F_1 \end{bmatrix}$$

For the two stations, these constants computed by means of formulas (9), p. 11 of Reference (12) are

$$\begin{array}{cc} \text{Pi C} & \text{Pi D} \\ \begin{bmatrix} -.41852302 & .85081668 & .31772545 \\ .66879650 & .52540705 & -.52598353 \\ .61445075 & .00764255 & .78891816 \end{bmatrix} & \begin{bmatrix} .78525690 & -.11693962 & -.60802691 \\ -.08762571 & -.99310808 & .07783372 \\ .61293828 & .00784068 & .79009189 \end{bmatrix} \end{array}$$

The direction cosines or above constants are now referred to the local coordinate system by multiplying the above matrices by the appropriate $A\Gamma$ matrix. The resulting sets of constants are

$$\begin{array}{cc} \text{Pi C} & \text{Pi D} \\ \begin{bmatrix} -.23270022 & -.90427548 & .26757506 \\ -.71597237 & .42687840 & .55241144 \\ .61375415 & .00778894 & .78945879 \end{bmatrix} & \begin{bmatrix} -.35513278 & .89608144 & .26630575 \\ .70642401 & .44382330 & -.55134923 \\ .61224651 & .00767742 & .79062965 \end{bmatrix} \end{array}$$

A convenient check on the numerical work is furnished by the relations associated with the direction cosines of three mutually perpendicular lines. Thus, the sums of the squares of the elements in any row or column must be equal to one, and the sums of the products of the corresponding elements of any two rows or columns must be equal to zero.

The direction of the ray from the camera to a flash point may be found using formulas (11) of Reference (12) to compute the standard coordinates

$$\xi = \frac{u}{w}$$

$$\eta = \frac{v}{w}$$

The azimuth angle K, measured from the positive X-axis in a clockwise direction is given by

$$\tan K = \frac{\eta}{\xi}$$

The elevation angle ϵ, is given by

$$\cot \epsilon = \sqrt{\xi^2 + \eta^2}$$

For the RC-7 flash we have the following values:

	Pi C	Pi D
l_x	.206052	.094452
l_y	.163614	.115106
u	.11457501	.10914691
v	.16297441	-.18588699
w	.31644561	.30203883
x	.36206857	.36136715
y	.51501555	-.61544070
tan K	1.42242545	-1.70308981
cot ϵ	.62955116	.71369004
K	$54^\circ\ 53'\ 30''.63$	$-59^\circ\ 34'\ 47''.65$
ϵ	$57^\circ\ 48'\ 26''.95$	$54^\circ\ 29'\ 05''.74$

The XYZ coordinates of the two stations are

	Pi C	Pi D
X	-1535.0080	-1533.7141
Y	-5452.0527	5452.1625
Z	54.8438	40.1300

The azimuth angle of the base line is $89^\circ\ 59'\ 35''.52$

The adjustment and triangulation is made, following the method outlined in Sec. 1, of Reference (6). α_a is the angle at Pi C and α_b at Pi D. The values of a and b are .00134937 and 10904.2153 meters respectively. Hence, we have the values

$$\alpha_a = 89^\circ\ 59'\ 35''.52 - K_c = 35^\circ\ 06'\ 04''.89$$
$$\alpha_b = 90\ \ 00\ \ 24.48 + K_D = 30\ \ 25\ \ 36.83$$
$$\epsilon_a = 57\ \ 48\ \ 26.95$$
$$\epsilon_b = 54\ \ 29\ \ 05.74$$
$$\alpha_a + \alpha_b = 65\ \ 31\ \ 41.72$$
$$d = \sin\alpha_b \tan\epsilon_a - \sin\alpha_a \tan\epsilon_b + a \sin(\alpha_a + \alpha_b)$$

$$\sin\alpha_a = .57502464 \qquad \cos\alpha_a = .81814$$
$$\sin\alpha_b = .50643861 \qquad \cos\alpha_b = .86228$$
$$\sin(\alpha_a+\alpha_b) = .91016567 \qquad \cos(\alpha_a+\alpha_b) = .41424$$
$$\tan\epsilon_a = 1.58843328 \qquad \cos^2\epsilon_a = .28384$$
$$\tan\epsilon_b = 1.40116849 \qquad \cos^2\epsilon_b = .33747$$
$$d = -.00003431$$

The coefficients of the condition equation are

$$a_1 = -1.1469$$
$$a_2 = 1.3691$$
$$a_3 = 1.7842$$
$$a_4 = -1.7039$$

Then $\left[aa\right] = 9.2765$, $K = 1.0738$ and the corrections to be applied to the measured angles are

$$v_{\alpha_a} = -0''.87$$
$$v_{\alpha_b} = 1.04$$
$$v_{\epsilon_a} = 1.36$$
$$v_{\epsilon_b} = -1.30$$

Adding these corrections, result in

$$\alpha_a + v_{\alpha_a} = 35^\circ\ 06'\ 04''.02$$
$$\alpha_b + v_{\alpha_b} = 30\ \ 25\ \ 37.87$$
$$\epsilon_a + v_a = 57\ \ 48\ \ 28.35$$
$$\epsilon_b + v_b = 54\ \ 29\ \ 04.44$$
$$\alpha_a + \alpha_b = 65\ \ 31\ \ 41.89$$

The functions of the angles needed to compute the coordinates of the flash point are

$$\sin \alpha_a = .57502120$$
$$\sin \alpha_b = .50644296$$
$$\sin (\alpha_a+\alpha_b) = .91016601$$
$$\tan \epsilon_a = 1.58845650$$
$$\tan \epsilon_b = 1.40114981$$

Then

$$\frac{b \sin \alpha_b}{\sin (\alpha_a+\alpha_b)} = 6067.4240$$
$$\frac{b \sin \alpha_a}{\sin (\alpha_a+\alpha_b)} = 6889.0234$$

and

$$Z_a = 9692.6829$$
$$Z_b = 9692.6838$$

The corrected azimuth angles measured from the + X axis are

Pi C	Pi D
$54^\circ\ 53'\ 31''.50$	$-59^\circ\ 34'\ 46''.61$

For Pi C:

$$X = \frac{b \sin \alpha_b}{\sin (\alpha_a + \alpha_b)} \cos K + X_c$$

$$Y = \frac{b \sin \alpha_b}{\sin (\alpha_a + \alpha_b)} \sin K + Y_c$$

and for Pi D:

$$X = \frac{b \sin \alpha_a}{\sin (\alpha_a + \alpha_b)} \cos K + X_D$$

$$Y = \frac{b \sin \alpha_a}{\sin (\alpha_a + \alpha_b)} \sin K + Y_D$$

and

	Pi C	Pi D
sin K =	.81807026	-.86233356
cos K =	.57511829	.50634062
X =	1954.4785	1954.4783
Y =	- 488.4736	- 488.4736

Averaging the results from Stations Pi C and Pi D, the coordinates of the RC-7 flash point are

$$X = 1954.478 \text{ m}$$
$$Y = -488.47 \text{ m}$$
$$Z = 9692.683 \text{ m}$$

Table 26 is a tabulation of XYZ coordinates resulting from triangulations by the two Askania cameras. The Z coordinates are not corrected for refraction. The space coordinates of the aircraft, at moment of firing and of the burst, after being corrected for refraction are shown in Table 27. The check-runs for 21 March M1/P2 and 10 April M3/P1-1 and M3/P1-2 are also tabulated in Table 26. A comparison between the original reductions and the check run reductions are an indication of differences to be expected from the two sets of independent plate measurements.

4.6 Space Coordinates of Aircraft at Time of Firing

A sample calculation for 7 May M1/P1 follows:

	(Meters) X	Y	Z
1. RC-7 flash	1954.478	-488.474	9692.683
2. Askanias (Height of Instruments)			1.00
3. Refraction Correction (Table 20)			- 1.42
4. Aircraft Velocity correction	.957		
5. Position of RC-7 Flash at Time Gun Fired	1955.435	-488.474	9692.263
6. Distance between RC-7 Flash and End of Gun Barrel	- .38	00	- .81
7. Position of End of Gun Barrel when Gun Fired	1955.055	-488.474	9691.453

The space coordinates of the RC-7 flash were obtained from the Askania cameras (Table 26). The aircraft velocity correction in X was computed from the time difference between the RC-7 flash and the gun firing (Table 25), and the average velocity of the aircraft. Corrections in the Y and Z directions were so small as to be negligible.

4.7 Space Coordinates of Burst

A sample calculation for 7 May M1/P1 follows:

	(Meters) X	Y	Z
1. Burst without refraction corrections	2424.414	-600.480	8043.853
2. Refraction correction			- 1.28
3. Burst	2424.414	-600.480	8042.573

The space coordinates of the burst were obtained from the Askania cameras (Table 26) and the refraction correction from 3.5.

4.8 Velocity of Aircraft

Since the time was not picked up until the first RC-7 flash occurred, the average velocity of the aircraft at any particular firing was computed from the reset and post-survey flashes following this event. Thus, the average velocity of the aircraft is determined from the survey flashes that occur up to approximately 3 seconds after the gun is fired. In view of the fact that the differences in times between the post-survey flashes are practically constant (0.5 sec.), and only the time between the reset and first post-survey flash differs from this 0.5 second value, the velocity of the aircraft was taken as the average of the velocities between the reset and post-survey flashes. A test calculation was made whereby the velocity was computed by weighting the velocities between flashes to the differences in times between the flashes. The results of the two methods agreed to within 1cm/sec., thus eliminating the need of any weighting.

A sample calculation is given of the aircraft velocity for 11 April M1/P3-1. From Tables 25 and 26 the ΔX, ΔY, ΔZ values between the reset flash and post-survey flash No. 10 are

ΔX	ΔY	ΔZ	Distance	Δt	Velocity (meters/sec.)
347.9599	16.3584	1.3968	348.3470	1.65023	211.0900
105.7541	5.1530	.4025	105.8803	.50062	211.4984
105.7038	5.4964	- .2347	105.8469	.49998	211.7022
105.5532	5.1004	- .1884	105.6765	.49993	211.3826
105.9001	5.3265	- .6714	106.0361	.50002	212.0637
105.6869	5.5317	- .3608	105.8322	.50005	211.6432
105.8262	5.4877	- .1786	105.9685	.49998	211.9456
105.6437	5.4823	-1.1120	105.7917	.49997	211.5961
106.0994	5.6466	- .8026	106.2526	.50001	212.5009
105.7137	5.5490	- .5486	105.8607	.50000	211.7213

Mean Velocity = 211.7144

(entered in Table 27)

A bias or constant error in settings made on the survey flashes will not affect the value of the aircraft velocity. The similarity in appearance of the photographed flashes result in very accurately measured distances, within approximately 5 microns on the plate. With a picture scale of 1:27,000, this is equivalent to 0.135 m in the air. Dividing this distance by the average difference in time between the flashes of 0.5 seconds, the aircraft velocity is determined to approximately 0.27 m/sec. If the aircraft is not accelerating or decelerating during these times a better velocity value is obtained by averaging. Because the aircraft velocity may not be constant its velocity should be determined with the help of the pre- and post-survey flashes. The requirement of aircraft velocity to ± 2.0 ft/sec. appears to have been satisfied.

4.9 Accuracy of Askania Results

The following tabulation is a sample of mean errors in the XYZ coordinates of the RC-7 flashes as triangulated by the Askania cameras. An example of the computations is also given.

Date	Mission/ Pass	Point Triangulated	m.e. (microns)	m.e. X	m.e. Y (meters)	m.e. Z
7 May	M1/P1	RC-7 Flash	±2.8	±.073	±.072	±.125
7 May	M1/P2	RC-7 Flash	±8.2	±.209	±.203	±.359
7 May	M1/P4	RC-7 Flash	±4.5	±.116	±.112	±.202

A Sample Calculation of Mean Errors in the Position of the RC-7 Flash for 7 May M1/P1 follows.

Coordinates of flash point from triangulation (4.5 (B)):

$$X = 1954.4784$$
$$Y = -488.4736$$
$$Z = 9692.6834$$

Coordinates of Cameras:

	Pi C	Pi D
X =	-1535.0080	-1533.7141
Y =	-5452.0527	-5452.1625
Z =	54.8438	40.1300

Observation Equations (see coefficients of Δx, Δy, Δz in Formulas 48, Reference (12))

ΔX	ΔY	ΔZ	$-\Delta l$
.0000118585	.0000254378	-.0000173942	.00000234
.0000289422	-.0000155616	.0000024645	.00000119
.0000114464	-.0000227961	-.0000181662	-.00000129
-.0000276427	-.0000147086	.0000009370	.00000122

Normal Equations:

ΔX	ΔY	ΔZ	L
$1873.4189 \times 10^{-12}$	-3.0806×10^{-12}	$-511.4360 \times 10^{-12}$	$-.13700158 \times 10^{-10}$
	$1625.2502 \times 10^{-12}$	$-\ 3.7821 \times 10^{-12}$	$-.52468625 \times 10^{-10}$
		639.5207×10^{-12}	$.19057645 \times 10^{-10}$

Inverse of Normal Equations:

68288×10^4	256×10^4	54613×10^4
256×10^4	61531×10^4	569×10^4
54613×10^4	569×10^4	200045×10^4

$$\Delta_X = -.0009$$
$$\Delta_Y = .0322$$
$$\Delta_Z = -.0303$$

Then, substituting in the observation equations:

$$v'_X = -.00000100$$
$$v'_Y = -.00000164$$
$$v''_X = .00000110$$
$$v''_Y = -.00000170$$

and $m = \pm .00000280 = \pm 2.8$ microns

The weighting coefficients from the inverse are

$$\sqrt{Q_X} = 26137$$

$$\sqrt{Q_Y} = 25599$$

$$\sqrt{Q_Z} = 44726$$

and

$$m_X = \pm .0732 \text{ m}$$

$$m_Y = \pm .0717 \text{ m}$$

$$m_Z = \pm .1252 \text{ m}$$

Since the geometry of the flashes in relation to the Askania cameras, and the mean errors of the plate measurements for all missions are practically the same, the requirement of determining the space position of the aircraft at the moment of firing and the space position of the burst to ± 2.0 ft. are satisfied.

5. VELOCITY CAMERAS

5.1 Projectile Velocity Determination

Four 35mm Robot type cameras (Fig. 12) were used to determine the muzzle velocity of the projectile. These velocity cameras were positioned (Figs. 13 and 14) fore and aft in the nose of the aircraft so as to form two baselines, each approximately 2.5 meters in length. The first pair, cameras Nos. 2 and 3 were so oriented that a picture of the projectile would be taken at time t_1, soon after leaving the end of the gun barrel. The second pair, cameras Nos. 1 and 4, were so oriented that a second exposure would be taken at time t_2, a fraction of a second later. The position of the projectile from each set of cameras can be computed(6) and since the elapsed time $(t_2 - t_1)$ between the exposures can be measured, the velocity of the projectile near the muzzle of the gun can be determined. Scale for the velocity camera system was essentially determined from the coordinates of the control points which were used in establishing the orientations of the velocity cameras. The baseline distances between the four velocity cameras were determined by direct measurements. These distances do not enforce the scale of the velocity camera system but were used to establish the $X_o\ Y_o\ Z_o$ positions of the four cameras. Small deviations of the $X_o\ Y_o\ Z_o$ positions from their true values were compensated for by specific camera orientation parameters α, ω, and κ .

Two exposures are taken by each camera (Fig. 15). The first exposure was taken over the calibration pit where the relative orientations of the cameras were determined by the use of control points. The second exposure was taken over the range soon after the gun was fired. The small circles (Fig. 15) show the fiducial marks, control points and the projectile that were measured on the 35mm velocity camera film.

5.2 Relative Stability of Velocity Cameras

A total of five 35mm Robot type cameras were mounted in the aircraft on an invar system. Camera No. 5 was used to check the stability of the system by photographing a cross mark positioned in a collimated light

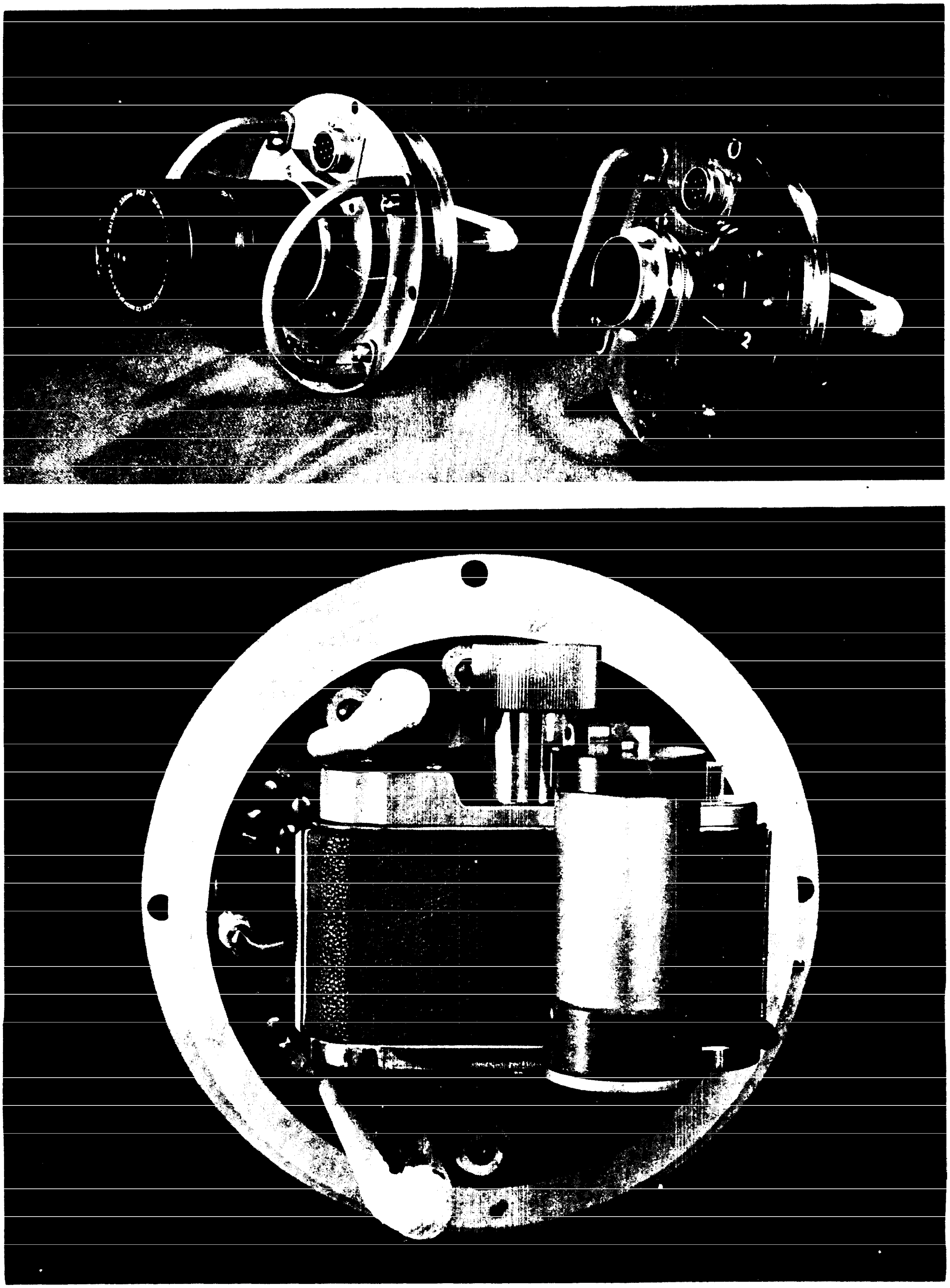


Fig. 12 Front and Rear View of a Velocity Camera

Fig. 13 Forward Velocity Camera Installation, Rear Velocity Camera and RC-7 Camera Installation

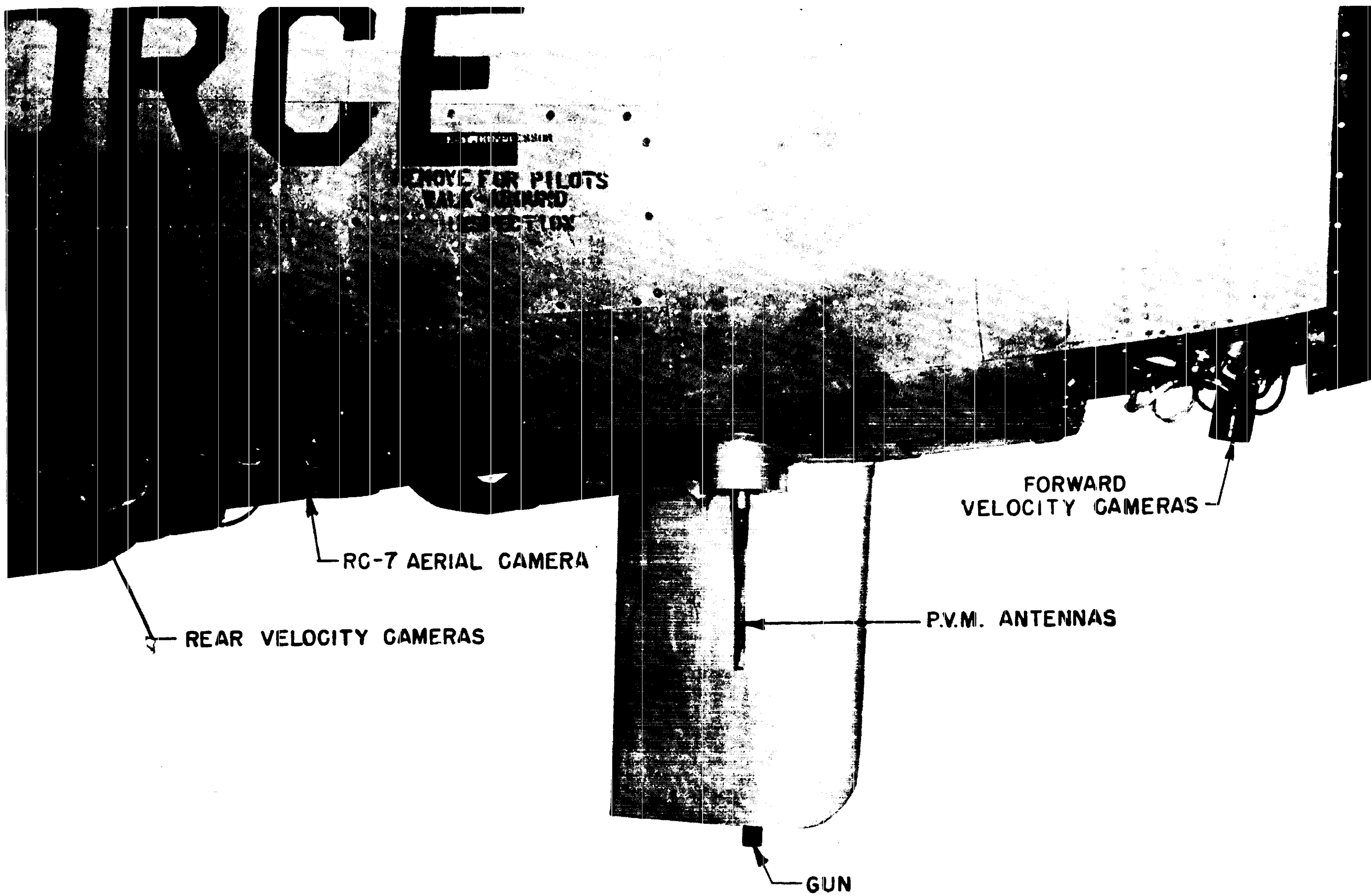


Fig. 14 F-89 Nose Installation

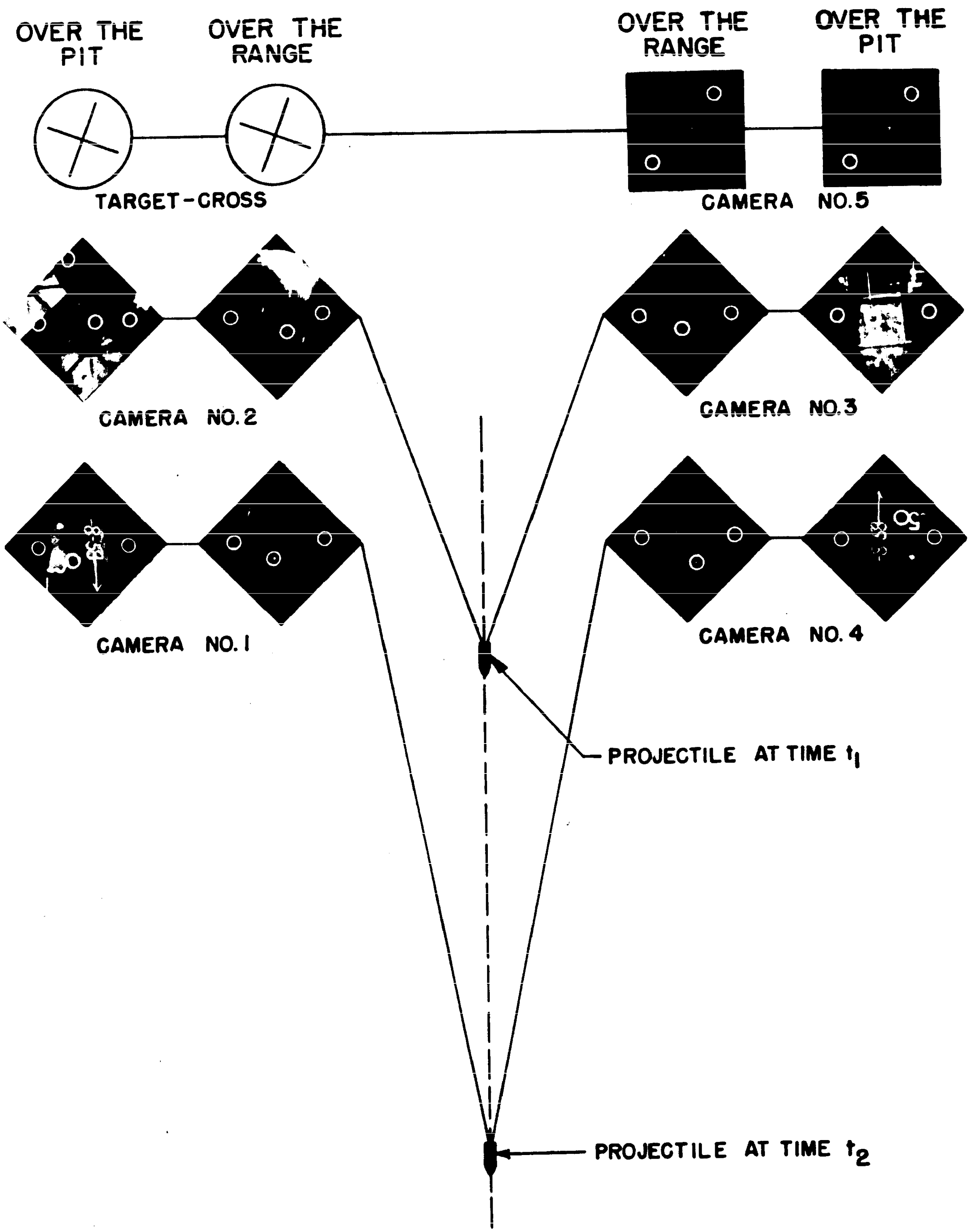


FIG. 15 - VELOCITY CAMERA EXPOSURES OVER CALIBRATION PIT AND RANGE

source. Any rotation in cameras Nos. 3 and 4 relative to cameras Nos. 1 and 2 (Fig. 16) will result in a change of position of the image of the cross relative to the fiducial marks in the two exposures made with camera No. 5.

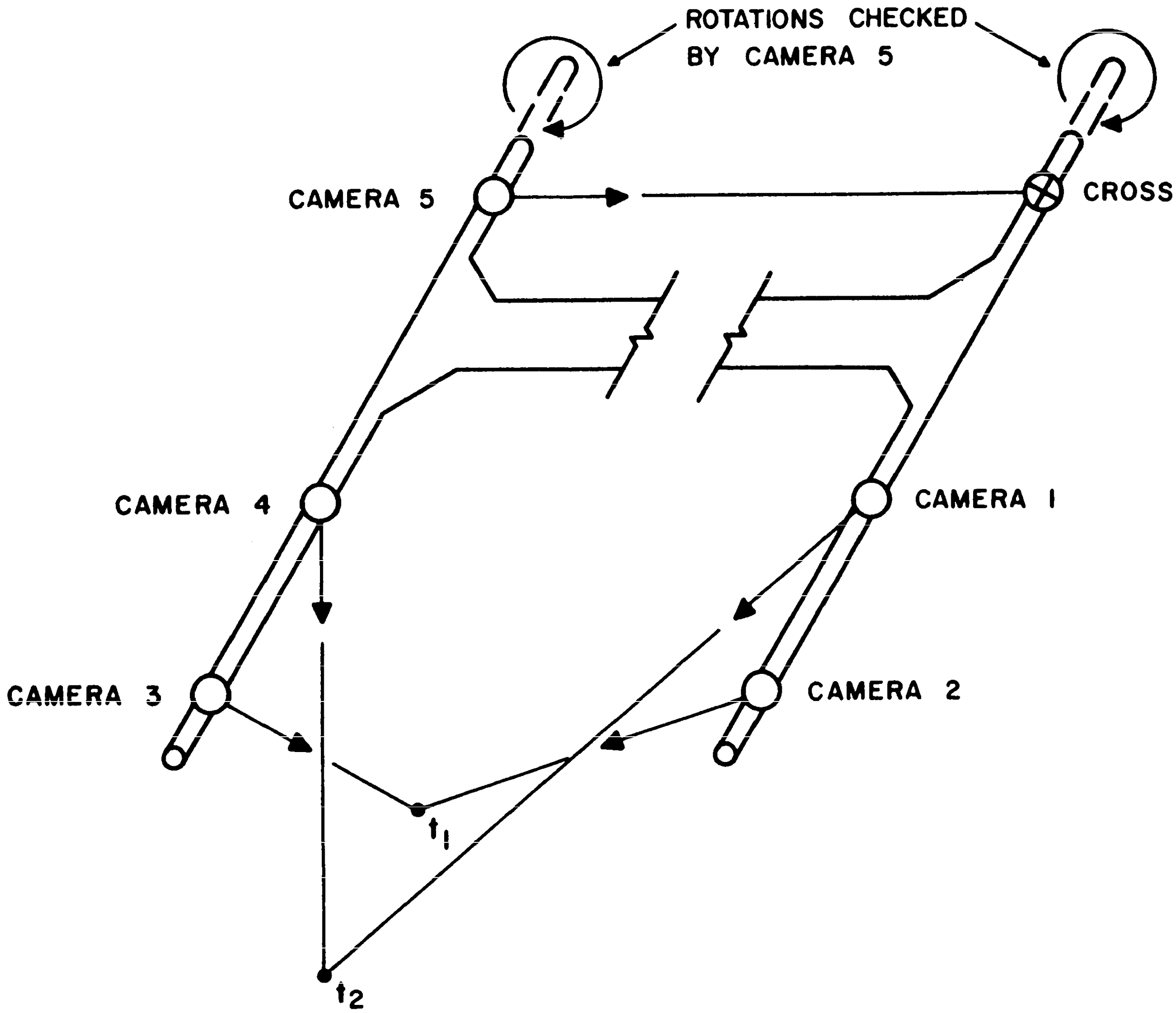


FIG. 16 - SCHEMATIC OF VELOCITY CAMERAS

THE SYSTEM PROVIDES FOR A STABILITY CHECK BETWEEN CONDITIONS AT TIMES OF CALIBRATION AND FIRING.

As an example of the method used to compute the relative stability of the velocity camera system, computations for 7 May M1/P1 follow.

CAMERA NO. 5

Film measurements while over the calibration pit:

	l_x (meters)	l_y
Fiducial Mark No. 1, denoted 001	.037956	.023865
Target Cross, denoted 002	.046259	.017328
Fiducial Mark No. 2, denoted 003	.051158	.010684

Film measurements for 7 May M1/P1 firing:

Fiducial Mark No. 1, denoted 101	.061536	.024516
Target Cross, denoted 102	.069838	.017996
Fiducial Mark No. 2, denoted 103	.074735	.011368

	Δl_x	Δl_y	Distance
003 - 001	.013202	-.013181	.0186556
103 - 101	.013199	-.013148	.0186302

$$\text{Scale factor} = \frac{.0186556}{.0186302} = 1.001363$$

During Calibration

	l_x (meters)	l_y	Distance
$\frac{003 + 001}{2}$ =	.044557	.017274	
002 =	.046259	.017328	
difference =	- .001702	- .000054	.0017029

During Mission

	l_x	l_y	Distance
$\frac{103 + 101}{2}$ =	.068135	.017942	
102 =	.069838	.017996	
difference =	- .001703	- .000054	.0017039

.0017039 x scale factor = .0017062

distance during calibration = .0017029

difference = .0000033

= 3.3 microns

focal length of camera No. 5 = 152, 000 microns

amount system moved in seconds of arc $= \frac{3.3 \times 206,265}{152,000} = 4.5''$

Distance between Target Cross and Camera No. 5 is approximately 2.3 m.

Amount moved in millimeters $= \frac{3.3 \times 2,300}{152,000} = .050$

Summary of stability checks:

Mission	Amount of Movement	
7 May M1/P1	4.5"	.050mm
7 May M1/P2	- 5.7"	- .064mm
7 May M1/P4	- 6.2"	- .069mm
7 May M1/P5	- 5.8"	- .065mm
8 May M3/P2	- 4.3"	- .048mm

The above results indicate that the camera orientations as obtained during the calibrations are little affected by any distortion of the invar system.

5.3 Calibration of Velocity Cameras

Because of the lack of any control points for the velocity camera orientations at the time of airborne firing, the following method was used.

Before a mission the camera orientations were determined from photographs taken over the calibration pit and were assumed to remain relatively fixed to each other until after the gun was fired.

The film measurements of the projectile from the photographs taken in the air were then evaluated in terms of the calibration parameters observed over the calibration pit. This was accomplished by computing the difference between fiducial marks and projectile from the photograph taken during the firing, applying a scale factor correction for film shrinkage and thus

interpolating the rays of the projectile into the calibration records. Using these reduced projectile film measurements, the coordinates of the projectile were determined.

The positions of the projectile at times t_1 and t_2 are in the calibration pit coordinate system. However, this is of no consequence since only a difference in coordinates of the projectile is required for a velocity computation.

An example of computing the reduced projectile film measurements for Camera No. 1 taken 7 May M1/P1 follows. To simplify the computation, the film when measured on a comparator is lined up so that the x-axis of the comparator will run through both fiducial marks.

<u>7 May M1/P1 Camera No. 1</u>

Film measurements while over the calibration pit:

	1_x (meters)	1_y
Fiducial No. 1 called 001	.281414	.300000
Fiducial No. 2 called 003	.300001	.300000
difference	.018587	.000000

distance between fiducial marks: .018587

Film measurements for 7 May M1/P1 firing:

	1_x (meters)	1_y
Fiducial mark No. 1 called 101	.281450	.300000
Fiducial mark No. 2 called 103	.300000	.300000
difference	.018550	.000000

distance between fiducial marks: .018550

$$\text{Scale factor} = \frac{.018587}{.018550} = 1.001995$$

The projectile film measurement from 7 May M1/P1 firing is denoted as 102, and computed projectile measurement reduced to film taken over the calibration pit is denoted as 102'. The film measurements are

	l_x (meters)	l_y
102	.289299	.303506
101	.281450	.300000
Δl_x =	.007849 Δl_y =	.003506
(Δl_x)(scale factor) =	.007865	
(Δl_y)(scale factor)=	.003513	
001	.281414	.300000
correction	.007865	.003513
102'	.289279	.303513

The absolute control points, one situated at the bottom of the calibration pit and the other one on the gun rod, were so arranged that they would be approximately in the same relative position with respect to the velocity cameras at the time of the calibration as the projectile was during the mission. If the control points and the projectile were exactly in the same relative position, any camera orientation obtained from the control points when used to triangulate the position of the projectile would produce coordinates for the projectile equal to the original coordinates of the control points. Inasmuch as the relative position of the control points and projectile are approximately the same, small biases in the individual camera orientation parameters have negligible effects on the position computations of the projectile.

5.4 Scale Factor

A scale factor was used in all velocity camera reductions. It was assumed that the difference in the distances between fiducial marks as measured during the calibration and mission was due to temperature changes in the camera and of the film. It is important to normalize this distance since all velocity camera photographs of the projectile were reduced after being projected onto the photograph taken during the calibration.

5.5 Camera Orientation Elements

The camera orientation elements for each velocity camera are α, ω, κ, X_o, Y_o, Z_o, c, x_p, y_p. Six of these nine orientation elements are either known or can be adequately assumed for the velocity cameras, or can be determined by independent measurements.

The three elements of interior orientation c, x_p and y_p are

c, the principal distance or focal lengths of the cameras which was given, and

x_p and y_p, the coordinates of the principal point were assumed to be zero.

The six elements of exterior orientation are X_o, Y_o, Z_o and α, ω, κ.

X_o, Y_o, Z_o, the coordinates of the center of projection for each velocity camera were determined by the following method. Distances were measured between the centers of projection of the RC-7 camera and the velocity cameras. The coordinates of the velocity cameras for each calibration could then be determined by algebraically adding these distances to the coordinates of the RC-7 camera as determined over the calibration pit.

α, ω, κ, the three rotational orientation elements, remain to be determined to complete the required nine camera orientation elements.

5.6 Methods of Determining the Velocity Camera Orientations

The number of absolute control points necessary for a unique solution, to determine the orientation elements of a single camera is n, where

$$2n = u$$

n = number of absolute control points

u = number of unknowns

with

u = 3 (the three rotational elements α, ω, κ)

$$n = 1\ 1/2$$

Only one absolute control point is available by which the velocity camera orientations can be determined, point No. 25 for cameras Nos. 1 and 4 and point No. 27 for cameras Nos. 2 and 3. It was therefore necessary to make use of relative control points applying the method of double point resection in space or to reduce the number of unknowns (u) by obtaining a value for one of the three rotational elements by some other means.

In the case of cameras Nos. 2 and 3, the value of κ was determined by direct measurement on the film. Since the tilt angle ω is small, the value for κ obtained in this manner is sufficiently accurate. Thus the minimum number of absolute control points will now be reduced to one. In this manner all camera orientations for velocity cameras No. 2 or No. 3 were made, following the methods outlined in Reference (4).

In the case of cameras Nos. 1 and 4 it was possible to make use of relative control points. Relative control points are points common to both cameras but whose coordinates are not known. It was not possible to make use of relative control points for cameras Nos. 2 or 3 because no common area was photographed.

All camera orientations for cameras Nos. 1 and 4 were made following the method as outlined in Reference (8) which determined the orientations of the two cameras simultaneously with the coordinates of the relative control points. The six unknowns to be determined for the two cameras are the three rotational angles α', ω', and κ' for camera No. 1 and α'', ω'', and κ'' for camera No. 4. Each point gives rise to 4 equations. Using the absolute control point No. 25 and 3 relative control points, 16 equations for the 15 unknowns of the solution are obtained, constituting a slight over-determination which acted as a check against coarse errors.

5.7 Number of Cameras Required for a Projectile Velocity Determination

It is preferred that all four velocity camera orientations be known in which case the XYZ coordinates of the projectile for times t_1 and t_2 were obtained by triangulation. However, in some cases because of poor quality

images of the projectile, camera malfunction, etc., only three or less of the four camera records were available. Muzzle velocity reductions were not made when less than three camera orientations were available or when camera No. 1 or No. 4 were missing. Any approximate method of solving the position of the projectile when camera No. 1 or No. 4 is missing could be critical because of the weak geometry inherent in the measuring set-up.

5.8 Projectile Determination when Two Camera Orientations are Known

When the projectile is at time t_1 and both camera No. 2 and No. 3 orientations are known, or at time t_2 when both camera No. 1 and No. 4 orientations are known, the projectile position is determined by the methods outlined in Reference (6). For an example of this method see 5.10.

5.9 Projectile Determination when either Camera No. 2 or No. 3 is Missing

When the camera orientation for either camera No. 2 or No. 3 was missing a close approximation to the position of the projectile was obtained by the intersection of the line extending through the gun barrel axis and the projection of the ray from one of the cameras to the projectile onto the plane containing the gun barrel axis and the center of projection of that camera. Thus the position of the projectile at time t_1 was determined by the intersection of the two corresponding lines. For an example of this method see 5.11.

5.10 Computations of the Coordinates of the Projectile by Triangulation from Two Known Camera Orientations

In this example the orientations of cameras Nos. 1 and 4 were determined by using one absolute and three relative control points. This example is for 7 May M1/P1.

The approximation values for α, ω, κ, were taken from a similar solution for another date. It would have been possible to use approximation values obtained from the geometry of the physical set-up of the cameras in which case several iterations would be required before the corrections to the approximations remained sufficiently unchanged. In this example no iterations were required.

The values for the center of projection of the velocity cameras were determined as follows:

The RC-7 camera center of projection values as determined over the calibration pit for the 7 May M1/P1 are (Table 15)

X_o = 96.4368 m Y_o = 96.7051 m Z_o = 5.6684 m

The measured distances between the RC-7 and velocity cameras Nos. 1 and 4 are

Camera No. 1	ΔX = 1.8054	ΔY = - 0.1100	ΔZ = 0.1968
Camera No. 4	ΔX = -0.5135	ΔY = - 0.0900	ΔZ = -0.0117

Applying these distances to the RC-7 values the centers of projection for the two velocity cameras are

Camera No. 1	X_o = 98.2422	Y_o = 96.5951	Z_o = 5.8652
Camera No. 4	X_o = 95.9233	Y_o = 96.6151	Z_o = 5.6567

The approximation values, coordinates of the cameras and elements of inner orientations are as follows:

		Camera No. 1	Camera No. 4
α^o	=	188° 51' 32.98"	163° 33' 14.83"
ω^o	=	00 32 26.87	00 07 33.53
κ^o	=	88 25 10.77	272 28 02.68
X_o	=	98.2416 meters	95.9227 meters
Y_o	=	96.5960	96.6160
Z_o	=	5.8652	5.6567
c	=	.152	.152
x_p	=	.290708	.390665
y_p	=	.300000	.400000

The coordinates of the absolute control point are (see Point No. 25, Table 1)

X = 97.5122
Y = 96.7380
Z = 0.6968

The plate measurements for the control points, both absolute and relative are

	Camera No. 1		Camera No. 4	
	l_x	l_y	l_x	l_y
abs. pt. No. 25	.287590	.302364	.394455	.396618
rel. pt. No. 4	.291942	.304349	.390202	.395147
rel. pt. No. 5	.291666	.294791	.389879	.404392
rel. pt. No. 6	.292120	.309032	.390337	.390668

The direction cosines from formulas (13) of Reference (8) are

	Camera No. 1	Camera No. 4
$A_1 = -\cos\alpha\cos\kappa + \sin\alpha\sin\omega\sin\kappa$	.02579661	.04066790
$B_1 = -\cos\omega\sin\kappa$	-.99957511	.99907045
$C_1 = \sin\alpha\cos\kappa + \cos\alpha\sin\omega\sin\kappa$	-.01356968	.01429505
$A_2 = -\cos\alpha\sin\kappa - \sin\alpha\sin\omega\cos\kappa$	.98773415	-.95887158
$B_2 = \cos\omega\cos\kappa$	.02757744	.04305104
$C_2 = \sin\alpha\sin\kappa - \cos\alpha\sin\omega\cos\kappa$	-.15369038	-.28293786
$D = \sin\alpha\cos\omega$	-.15399929	.28310886
$E = \sin\omega$	.00943855	.00219877
$F = \cos\alpha\cos\omega$	-.98802587	-.95908526

The approximation values for the three relative points, computed by formulas (56) and (57) of Reference (8) are

Point No.	X	Y	Z
4	97.5806	96.6048	+0.6366
5	97.2454	96.6065	+0.6452
6	97.7442	96.6029	+0.6413

The coefficients of the observation equations are given by formulas (40) and (41) of Reference (8). The observation equations are

OBSERVATION EQUATIONS

Pt. No.	$\frac{\partial F}{\partial X} \Delta X_4$	$+ \frac{\partial F}{\partial Y} \Delta Y_4$	$+ \frac{\partial F}{\partial Z} \Delta Z_4$	$+ \frac{\partial F}{\partial X} \Delta X_5$	$+ \frac{\partial F}{\partial Y} \Delta Y_5$	$+ \frac{\partial F}{\partial Z} \Delta Z_5$	$+ \frac{\partial F}{\partial X} \Delta X_6$	$+ \frac{\partial F}{\partial Y} \Delta Y_6$	$+ \frac{\partial F}{\partial Z} \Delta Z_6$
25	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
4	+.00078	-.02884	-.00015	0	0	0	0	0	0
4	+.028626	+.00079	-.00362	0	0	0	0	0	0
4	+.00119	+.02874	+.00033	0	0	0	0	0	0
4	-.02732	+.00124	-.00902	0	0	0	0	0	0
5	0	0	0	+.00077	-.02861	-.00021	0	0	0
5	0	0	0	+.02812	+.00080	-.00537	0	0	0
5	0	0	0	+.00124	+.02931	+.00028	0	0	0
5	0	0	0	-.02837	+.00126	-.00749	0	0	0
6	0	0	0	0	0	0	+.00079	-.02901	-.00012
6	0	0	0	0	0	0	+.02893	+.00078	-.00276
6	0	0	0	0	0	0	+.00118	+.02851	+.00036
6	0	0	0	0	0	0	-.02687	+.00123	-.00976

OBSERVATIONS EQUATIONS

Pt. No.	$+\frac{\partial F}{\partial \alpha'} \Delta\alpha'$	$+\frac{\partial F}{\partial \omega'} \Delta\omega'$	$+\frac{\partial F}{\partial \kappa'} \Delta\kappa'$	$+\frac{\partial F}{\partial \alpha''} \Delta\alpha''$	$+\frac{\partial F}{\partial \omega''} \Delta\omega''$	$+\frac{\partial F}{\partial \kappa''} \Delta\kappa''$	$-\Delta l$	$= 0$
25	+.00413	+.15199	-.00225	0	0	0	+.00045269	= 0
25	+.15194	-.00423	-.00267	0	0	0	-.00011533	= 0
25	0	0	0	+.00663	-.15193	+.00335	+.00038921	= 0
25	0	0	0	-.15193	-.00647	+.00340	+.00002716	= 0
4	+.00419	+.15195	-.00434	0	0	0	+.00004097	= 0
4	+.15207	-.00416	+.00127	0	0	0	-.00001034	= 0
4	0	0	0	+.00654	-.15186	+.00487	+.00004145	= 0
4	0	0	0	-.15201	-.00656	-.00042	-.00001447	= 0
5	+.00421	+.15195	+.00521	0	0	0	+.00000717	= 0
5	+.15212	-.00423	+.00097	0	0	0	+.00000043	= 0
5	0	0	0	+.00656	-.15186	-.00440	+.00000954	= 0
5	0	0	0	-.15199	-.00652	-.00078	+.00000448	= 0
6	+.00419	+.15195	-.00903	0	0	0	+.00004596	= 0
6	+.15249	-.00412	+.00146	0	0	0	-.00000196	= 0
6	0	0	0	+.00654	-.15186	+.00935	+.00004739	= 0
6	0	0	0	-.15243	-.00659	-.00028	-.00001482	= 0

The solution of the normal equations formed from these observation equations yielded the following values:

$$\begin{aligned}
\Delta X_4 &= -0.0022 \\
\Delta Y_4 &= -0.0142 \\
\Delta Z_4 &= +0.0046 \\
\Delta X_5 &= -0.0018 \\
\Delta Y_5 &= -0.0165 \\
\Delta Z_5 &= +0.0071 \\
\Delta X_6 &= -0.0026 \\
\Delta Y_6 &= -0.0132 \\
\Delta Z_6 &= +0.0052 \\
\Delta\alpha' &= +0.0006131 \text{ rad.} = +02' \; 06.''47 \\
\Delta\omega' &= -0.00304264 \text{ rad.} = -10 \; 27.59 \\
\Delta\kappa' &= -0.00347670 \text{ rad.} = -11 \; 57.12 \\
\Delta\alpha'' &= +0.00006063 \text{ rad.} = +00 \; 12.51 \\
\Delta\omega'' &= -0.00279898 \text{ rad.} = -09 \; 37.33 \\
\Delta\kappa'' &= -0.01068718 \text{ rad.} = -36 \; 44.39
\end{aligned}$$

Due to the relatively small Δ corrections, one iteration cycle was considered sufficient.

Adding these values to the approximation values, the following elements for the relative orientation are

		Camera No. 1	Camera No. 4
$\alpha^o + \Delta\alpha$	=	$188^o \; 53' \; 39.''45$	$163^o \; 33' \; 27.''34$
$\omega^o + \Delta\omega$	=	00 21 59.28	- 00 02 03.80
$\kappa^o + \Delta\kappa$	=	88 13 13.65	271 51 18.31
c	=	.152	.152
x_p	=	.290708	.390665
y_p	=	.300000	.400000

Using this relative orientation and the measured coordinates of the projectile the azimuth and depression angles of the projectile at each of the two cameras was computed. Introducing the standard coordinates of the projectile, the coordinates at unit distance from the point of projection are

$$\zeta = \frac{(-1)u}{w}$$

$$\eta = \frac{(-1)v}{w}$$

From the relative orientation by means of formulas (9), Reference (12) the constants are

	Camera No. 1	Camera No. 4
A_1	.02969203	.03121765
B_1	-.99949726	.99947572
C_1	-.01111736	.00858750
A_2	.98752949	-.95859658
B_2	.03105323	.03237169
C_2	-.15434116	-.28292167
D	-.15460880	.28305133
E	.00639601	-.00060020
F	-.98795506	-.95910458

The quantities u, v and w are computed by means of formulas (11), Reference (12) making use of the plate measurements of the projectile:

	Camera No. 1	Camera No. 4
l_x	.289279	.392547
l_y	.303513	.404200
u	-.02005874	.03907040
v	.00248158	.00191169
w	-.15069795	-.14695248

From these values, the standard coordinates of the projectiles are

	Camera No. 1	Camera No. 4
ξ	-.13320758	.26576969
η	.01665319	.01310419

Then, the azimuth angle K (measured from +X clockwise) is $\tan^{-1} \eta/\xi$ and the depression angle ϵ, is $\cot^{-1} \sqrt{\xi^2 + \eta^2}$. Their values are as follows:

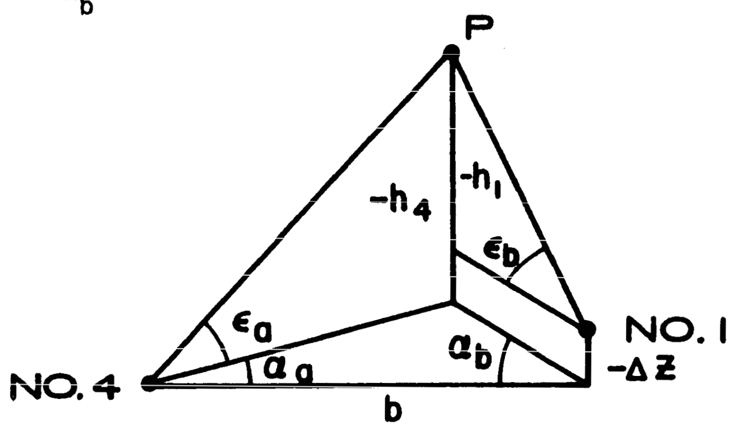
	Camera No. 1	Camera No. 4
tan K	-.12501681	.04930658
K	$172^\circ\ 52'\ 26''.53$	$2^\circ\ 49'\ 21''.98$
cot ε	.13424451	.26609256
ε	$82^\circ\ 21'\ 14''.64$	$75^\circ\ 05'\ 57''.47$

The intersection of these 2 rays is found by a least squares adjustment as outlined in Reference (6). From the XYZ coordinates of the cameras, the azimuth of the base line is determined. It is $179^\circ\ 30'\ 21''.06$ and the angles between the ray and the base line at each of the cameras is as follows:

At camera No. 1: $179^\circ\ 30'\ 21''.06 - K = 6^\circ\ 37'\ 54''.53$

At camera No. 4: $00\ \ 29\ \ 38.94 + K = 3\ \ 19\ \ 00.92$

From the following diagram, it can be seen that α_a is the angle at camera No. 4 and α_b at camera No. 1.



Hence

$$\begin{aligned}
\alpha_a &= 3^\circ\ 19'\ 00''.92 \\
\epsilon_a &= 75\ \ 05\ \ 57.47 \\
\alpha_b &= 6\ \ 37\ \ 54.53 \\
\epsilon_b &= 82\ \ 21\ \ 14.64 \\
\alpha_a + \alpha_b &= 9\ \ 56\ \ 55.45
\end{aligned}$$

$\sin \alpha_a = .05785888$ $\quad \cos \alpha_a = .99832$

$\sin \alpha_b = .11548871$ $\quad \cos \alpha_b = .99331$

$\sin (\alpha_a + \alpha_b) = .17276698$ $\quad \cos (\alpha_a + \alpha_b) = .98496$

$\tan \epsilon_a = 3.7580908$ $\quad \cos \epsilon_a = .25742$

$\tan \epsilon_b = 7.4490928$ $\quad \cos \epsilon_b = .13305$

$a = .08990944$

and from p. 8 Reference (6)

$$d = \sin \alpha_b \tan \epsilon_a - \sin \alpha_a \tan \epsilon_b + a \sin (\alpha_a + \alpha_b)$$

$$d = .01855427 \qquad d' = \rho \cdot d = (206265)(.01855427)$$

The coefficients of the condition equation (3) p. 8 Reference (6) are

$$a_1 = -\left[\cos \alpha_a \tan \epsilon_b - a \cos (\alpha_a + \alpha_b)\right] = -7.3480$$

$$a_2 = \left[\cos \alpha_b \tan \epsilon_a + a \cos (\alpha_a + \alpha_b)\right] = 3.8215$$

$$a_3 = \sin \alpha_b \sec^2 \epsilon_a = 1.7430$$

$$a_4 = - \sin \alpha_a \sec^2 \epsilon_b = -3.2689$$

and equation (3) becomes

$$-7.3480\, v\alpha_a + 3.8215\, v\alpha_b + 1.7430\, v\epsilon_a - 3.2689\, v\epsilon_b + 3827.0965 = 0$$

The corrections $v\alpha_a$, $v\alpha_b$, $v\epsilon_a$, $v\epsilon_b$, are computed by means of equations (5) Reference (6) where $K = - \frac{d'}{[aa]}$

$K = -39.7957$, where $[aa] = 82.2580$

$a_1K = v\alpha_a = 5' \; 41.''61$

$a_2K = v\alpha_b = -2' \; 57.''66$ $\qquad$ Formulas (5) p. 10 Reference (6)

$a_3K = v\epsilon_a = -1' \; 21.''03$

$a_4K = v\epsilon_b = 2' \; 31.''97$

Adding these corrections to the angles:

$$\alpha_a + v\alpha_a = 3^\circ\ 24'\ 42''.53$$

$$\alpha_b + v\alpha_b = 6\ \ 34\ \ 56.87$$

$$\epsilon_a + v\epsilon_a = 75\ \ 04\ \ 36.44$$

$$\epsilon_b + v\epsilon_b = 82\ \ 23\ \ 46.61$$

and $\alpha_a + v\alpha_a + \alpha_b + v\alpha_b = 9^\circ\ 53'\ 34''.19$

From the formulas on p. 7 Reference (6) and the figure, it follows that

$$Z = -\frac{b \sin \alpha_b \tan \epsilon_a}{\sin (\alpha_a + \alpha_b)} + Z_4 = -\frac{b \sin \alpha_a \tan \epsilon_b}{\sin (\alpha_a + \alpha_b)} + Z_1$$

where $b = 2.3190$ m

and

$$\sin \alpha_a = .05951221 \qquad Z_4 = 5.6567$$
$$\sin \alpha_b = .11463311 \qquad Z_1 = 5.8652$$
$$\tan \epsilon_a = 3.7521585$$
$$\tan \epsilon_b = 7.4909419$$
$$\sin (\alpha_a + \alpha_b) = .17354982$$

Hence

$$Z_a = -0.0905$$
$$Z_b = -0.0916$$

To find the X and Y coordinates of the projectile, the corrected base line angles are converted back into azimuth angles:

Camera No. 1	Camera No. 2
$172^\circ\ 55'\ 24''.19$	$2^\circ\ 55'\ 03''.59$

Then for Camera No. 1:

$$X = \frac{b \sin \alpha_a}{\sin (\alpha_a + \alpha_b)} \cos K + X_1$$

$$Y = \frac{b \sin \alpha_a}{\sin (\alpha_a + \alpha_b)} \sin K + Y_1$$

and similarly, for Camera No. 4:

$$X = \frac{b \sin \alpha_b}{\sin (\alpha_a + \alpha_b)} \cos K + X_4$$

$$Y = \frac{b \sin \alpha_b}{\sin (\alpha_a + \alpha_b)} \sin K + Y_4$$

where

$$\frac{b \sin \alpha_b}{\sin (\alpha_a + \alpha_b)} = 0.7952$$

$$\frac{b \sin \alpha_a}{\sin (\alpha_a + \alpha_b)} = 1.5317$$

Then

$$\sin K_1 = .12319643$$
$$\cos K_1 = -.99238230$$
$$\sin K_4 = .05090084$$
$$\cos K_4 = .99873712$$
$$X_1 = 98.2422$$
$$Y_1 = 96.5951$$
$$X_4 = 95.9233$$
$$Y_4 = 96.6151$$

with the result that

$X_a = 97.4531$ m $\qquad X_b = 97.4531$ m

$Y_a = 96.6931$ m $\qquad Y_b = 96.6931$ m

5.11 Computations of the Coordinates of the Projectile When One of the Two-Camera Orientations is Missing

In this example the orientation of camera No. 3 was missing. It was possible however to determine an approximation to the coordinates of the projectile by using the methods outlined under 5.9. The following example is for 7 May M1/P1.

The coordinates of points Nos. 26 and 27, which simulate the gun axis, and of camera No. 2 are

	X	Y	Z
Camera No. 2	98.2645	96.7760	5.8525
Control Pt. 26	97.3642	96.6982	5.0360
Control Pt. 27	97.3795	96.7016	4.1840

The equation of a plane containing these three points is

$$.06906X - .77955Y - .00187Z + 68.6665 = 0 \qquad (1)$$

The equation of a line through points Nos. 26 and 27 in the two point form is

$$\frac{X - 97.3642}{0.0153} = \frac{Y - 96.6982}{0.0034} = \frac{Z - 5.0360}{-0.8526} \qquad (2)$$

The direction of the ray from the camera to the projectile was determined from the camera data. Since there is only one control point it was necessary to find the value of κ by direct measurement on the film. κ was found to be 54.7 grad.

From the single camera orientation program, the values of α and ω were

$$\alpha = 231.04310 \text{ grad} = 207^\circ\ 56'\ 19.''64$$
$$\omega = -.00288388 \text{ grad} = -\ 00^\circ\ 00'\ 09.''34$$

where

$$\kappa = 54.7 \text{ grad} = 49^\circ\ 13'\ 48''.00$$
$$c = .050$$
$$x_p = .047612$$
$$y_p = .075485$$

Knowing the coordinates of two points on the ray to the projectile, the equation of a plane containing these two points and being perpendicular to the plane defined by equation (1) may be written. These two equations define a line which is the projection of the ray on the plane containing the camera and the two points on the line through the gun. The intersection of the two lines will give the coordinates of the projectile. Knowing the coordinates of one point on the ray, i.e. the coordinates of the camera and taking an arbitrary value of Z, by means of formulas (11) and (12) of Reference (10) the coordinates of another point on this ray may be found. From the orientations, the constants of formulas (9), Reference (12) are

$$A_1 = .57692936$$
$$B_1 = -.75733708$$
$$C_1 = -.30592981$$
$$A_2 = .66905456$$
$$B_2 = .65302415$$
$$C_2 = -.35485976$$
$$D_1 = -.46852801$$
$$E_1 = -.00004528$$
$$F_1 = -.88344864$$

The plate measurements of the projectile are

$$l_x = .050649$$
$$l_y = .075683$$

Then, from formulas (11) of Reference (12)

$$u = -.02154179$$
$$v = -.00217300$$
$$w = -.04517180$$

Letting Z = 3, from formulas (12) of Reference (12)

$(z) = 3 - 5.8525 = -2.8525$ and

$$X = \frac{(Z)u}{w} + X_o = 96.9042$$

$$Y = \frac{(Z)v}{w} + Y_o = 96.6388$$

Combining the coordinates of the two points on the ray and the condition of perpendicularity to the plane defined by equation (1) the equation of the plane through the ray is

$$2.22341X + 0.19953Y - 1.069908Z - 231.53072 = 0 \quad (3)$$

eliminating Y from the equations (1) and (3)

$$2.24109X - 1.07038Z = 213.95516$$

and from (2)

$$.8520X + 0.0153Z = 83.03135$$

$$X = 97.3827$$

$$Z = 4.0063$$

Then from (2)

$$Y = 96.7023$$

As a check, these coordinates were substituted in all three equations and the equations were satisfied within the limits of the accuracy of the computations.

5.12 <u>Computations of the Projectile Velocity. (7 May M1/P1)</u>

Coordinates of projectile at time t_2, computed from cameras Nos. 1 and 4 are

$$X = 97.4531$$

$$Y = 96.6931$$

$$Z = -0.0910$$

Coordinates of projectile at t_1, computed from camera No. 2 (camera No. 3 missing) are

$$X = 97.3827$$
$$Y = 96.7023$$
$$Z = 4.0063$$

The distance between the projectile time at t_1 and t_2 is

$$\sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} = 4.0979 \text{ m}$$

The projectile velocity will then be

$$\frac{4.0979}{.004194} = 977.1 \text{ m/sec.} = 3205 \text{ ft/sec.}$$

A tabulation (Table 28) of all projectile velocities that could be reduced by the velocity cameras is listed. Also listed are the values obtained by the PVM system.

5.13 Discussion of Projectile Velocity Results

During the planning stages of the project it was decided that projectile velocities would be determined by the electronic PVM method and checked by an optical method (Velocity Cameras).

It is believed that results obtained optically, while having a larger spread are closer to the true projectile velocity, whereas results by the electronic method are in themselves more consistent, but possess a constant or bias error. This conclusion was based on tests of the PVM system made at the Aberdeen Proving Ground. In these tests, comparisons between an optical and PVM system were made using a ground-based optical system which was accurately controlled. The results of 20 simultaneous observations showed a 4 ft/sec $\pm$ 1 ft/sec smaller velocity for the PVM system over the 2580 to 2640 ft/sec velocity range.

During the four airborne observations at EAFB a comparison of the two systems resulted in a 17.5 ft/sec $\pm$ 3.5 ft/sec smaller velocity for the PVM system in the 3050 to 3200 ft/sec range. It should be noted that this result was obtained from a small sample.

PVM projectile velocity data were transmitted by radio to the ground stations where they were recorded. In cases where the aircraft was not vertically above the recording stations at the moment of firing, a data error could exist due to a secondary Doppler effect resulting from motion of the aircraft. However, because of the relatively small aircraft velocity, this error proved to be negligible.

Assuming the PVM method to be good except for a constant error, the mean error of a single determination by the optical method may be computed.

Date	Difference (Opt.-PVM)	Difference ϵ (Opt.-PVM) (17.5 bias removed)
7 May 1/1	+ 16 ft/sec	-1.5 ft/sec
7 May 1/2	+ 10	-7.5
7 May 1/5	+ 27	+9.5
8 May 3/2	+ 17	-0.5

Mean = +17.5 ft/sec $\pm$ 3.5 ft/sec

The mean error of a single difference is approximately $\pm$ 2 m/sec, which, using an average time of .0042 seconds between times t_1 and t_2, amounts to an error in distance of 8mm.

In regard to the optical method, three factors that could contribute to this 8mm value are

(1) An incorrect time difference Δt between the two micro-flashes of approximately 9 micro-seconds, the approximate projectile velocity being 950m/sec.

Checks made at Aberdeen Proving Ground, Maryland showed Δt to be accurate to 1.2 $\pm$ 0.5 micro-seconds.

(2) Incorrect velocity camera orientations.

The reliability of the camera orientations is assured by the small mean errors of the control point plate measurements which, for most reductions, were less than 5 microns.

(3) Film measurements of the projectile.

An error of 8mm in the distance which the projectile travels between time t_1 and t_2 could, because of the geometry of the velocity camera system, be attributed to errors of approximately 7.5mm from camera No. 1 and No. 4 and approximately 2.5mm from cameras No. 2 and No. 3.

The errors in the projectile plate measurements that result in the 7.5mm and 2.5mm values may be computed from Fig. 17. Cameras Nos 2 and 4 are not shown and the dimensions listed are approximate values.

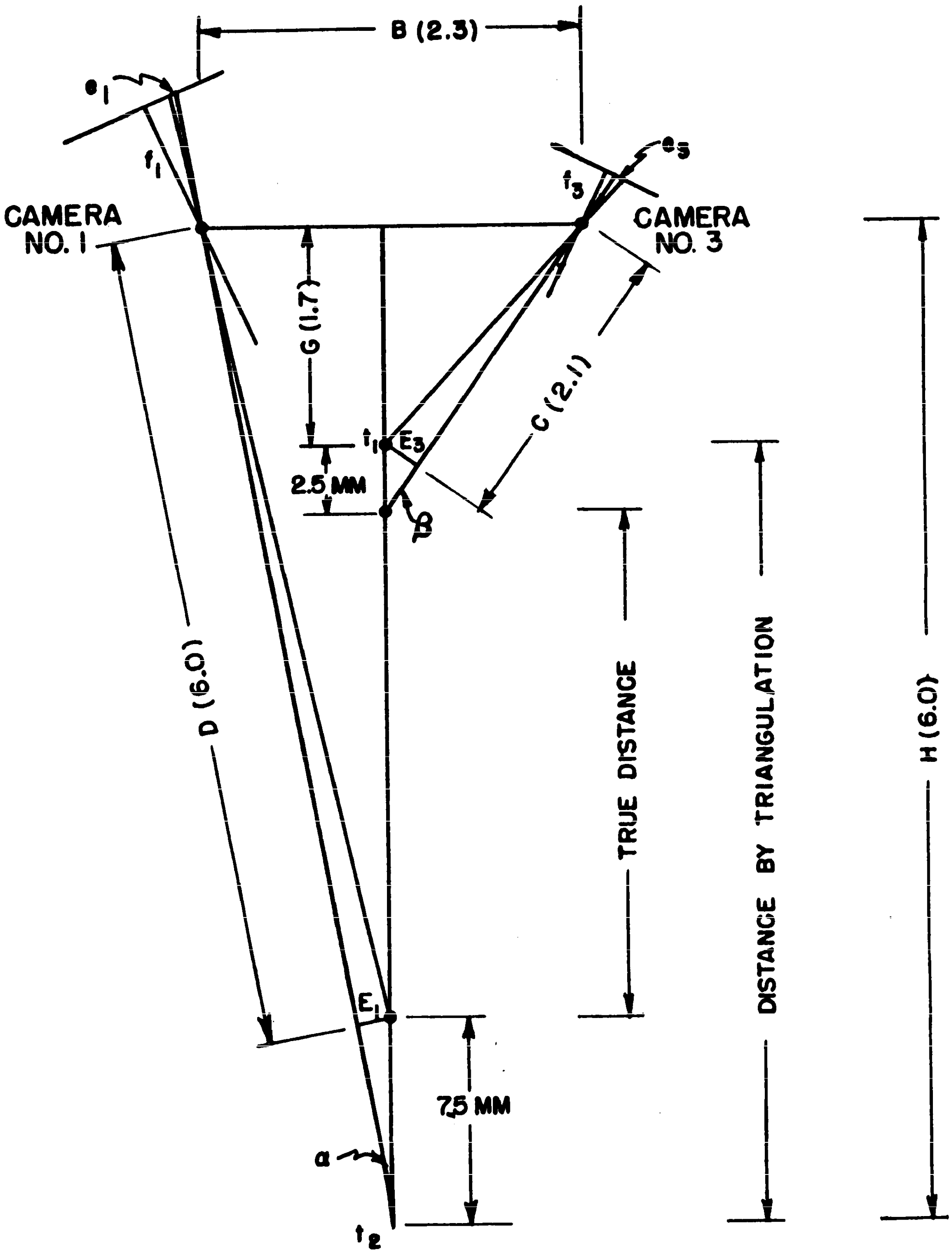


FIG. 17 - ERRORS IN PROJECTILE VELOCITY INDUCED BY IDENTIFICATION ERRORS IN PROJECTILE FILM MEASUREMENTS

$$\tan \alpha = \frac{B/2}{H} = \frac{E_1}{7.5\text{mm}}$$

$$E_1 = \frac{7.5(B/2)}{H} = \frac{7.5\frac{2.3}{2}}{6} = 1.44\text{mm}$$

$$\frac{e_1}{f_1} = \frac{E_1}{D}$$

$$e_1 = \frac{f_1E_1}{D} = \frac{152{,}000 \times 1.44}{6000} = \underline{\underline{36 \text{ microns}}}$$

$$\tan \beta = \frac{B/2}{G} = \frac{E_3}{2.5\text{mm}}$$

$$E_3 = \frac{3(B/2)}{G} = \frac{2.5 \times \frac{2.3}{2}}{1.7} = 1.69\text{mm}$$

$$\frac{e_3}{f_3} = \frac{E_3}{C}$$

$$e_3 = \frac{f_3E_3}{C} = \frac{50{,}000 \times 1.69}{2100} = \underline{\underline{40 \text{ microns}}}$$

The above computations indicate that an error of 8 millimeters in the distance the projectile travels from times t_1 to t_2 could be the result of film measuring errors (e_1 and e_3) between 35 and 40 microns on the projectile image.

e_1 and e_3 are not setting or measuring errors but are identification errors. Repeatability of settings made on the projectile image was within 10 microns. The size of the projectile image varies from approximately 600 to 1000 microns in diameter, and in some cases appears elliptically shaped. The possibility of a bias in the identification should not be ignored. Again it is noted that this 35 to 40 micron error was based on a very small sample of four measurements.

5.14 Final Projectile Velocities

If the results as determined by the four velocity cameras are used and the PVM values are considered good except for the 17.5 ft/sec bias, the final projectile velocities are tabulated in Table 29.

The projectile velocity determinations in common for the optical and electronic methods comprised too small a sample of measurements to determine with sufficient accuracy the difference between these methods. It is therefore doubtful that the projectile velocity was determined to $\pm$ 2 ft/sec.

If the bias due to the PVM method is to be determined by comparisons with airborne velocity cameras a large number of measurements should be made. In this way identification errors made when measuring the projectile image on the film may be averaged out. Also, to simplify the camera orientation reductions, at least three absolute control points should be photographed from all four velocity cameras.

However, a better method of obtaining the projectile velocities would be either to determine the PVM bias by comparisons made on the ground with a controlled optical system, or to determine and eliminate the cause of bias in the PVM system.

CONCLUSIONS

Calibration Pit

1. To check the relative positions of the twenty-five control points in the calibration pit, six widely separated control points should be observed before each RC-7 camera exposure over the calibration pit is made.

2. Unless the check survey on the six control points indicates a change in their relative positions a complete pit calibration on all twenty-five control points is not necessary.

3. Directions to the four stations of the quadrilateral and to the two control points simulating the center line of the gun barrel can be made to within 2 seconds of arc, and directions to the twenty-five pit control points can be made to within 5 seconds of arc. This would insure a sufficiently accurate determination of the relation between the gun and the RC-7 camera.

RC-7 Camera

1. Periodic cone calibrations would make it possible to observe any changes of metric significance in this basic measuring instrument.

The RC-7 cone should be calibrated by taking star exposures at the beginning of each group of firings. The calibrated focal length and distortion curve thus determined would then be used in camera orientation reductions over the range.

Lens distortion corrections should be determined simultaneously with the RC-7 camera orientation reductions for all plates taken over the calibration pit.

2. Until the problem is solved of maintaining with sufficient accuracy the interior orientation of the RC-7 the elements $X_oY_oZ_o$ for the RC-7 camera orientations over the range should be enforced from the position of the RC-7 camera as triangulated by the Askania cameras.

3. Results from this group of firings indicate that the requirement of determining the direction of the gun during the time of firings to within 1 mil was satisfied.

Askania Cameras

1. Since a systematic error in the orientation of the U.S.C. and G.S. Triangulation Net may exist, the flashes as triangulated by the two ground based Askania cameras will not necessarily be compatible with the range control points or the RC-7 camera results, as determined from these range control points (page 11, Reference (9)). A rigorous method is suggested whereby the flashes can be triangulated by using three ground based cameras following the method outlined in Reference (9). The computed coordinates of the flashes and the RC-7 camera results will then be reduced to the same local coordinate system.

2. The requirements of determining the relative space coordinates of the aircraft and the burst to $\pm$ 2.0 feet and the velocity of the aircraft to $\pm$ 2.0 ft/sec have been satisfied.

Velocity Cameras

1. The constant differences between the Projectile Velocity Meter (PVM) and velocity camera systems, if determined from airborne firings, should be based on a large number of comparisons. This constant difference could be much more readily obtained by ground-based comparisons where the optical method can be very accurately controlled. In either case the cause of a PVM bias should be understood and removed if possible.

2. The camera orientations of each of the four velocity cameras should be determined from at least three absolute control points.

3. Scale for the velocity cameras is the result of the computed coordinates of control points No. 27 on the gun rod and No. 25 in the pit. This scale will be adequately determined if directions to these points are known within five seconds of arc.

4. Results from camera No. 5 indicated that the stability of the invar bar system was greater than required.

5. It is doubtful that the requirement of determining the projectile velocity to $\pm$ 2.0 ft/sec for this small sample of rounds was completely satisfied. However, a sample of approximately 15 rounds, obtained with suitable calibration, would be sufficient to meet this requirement.

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MYRON W. LAWRENCE

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TABLE 1

XYZ COORDINATES FROM DIFFERENT PIT CALIBRATIONS
(Meters)

Date of Calibration	Cal. No.		Control Point Number 9	15	17	19	23	25
28 Feb 1958	07	X	96.3812	95.1599	94.6696	96.3590	98.0693	97.5126
		Y	98.2522	96.7266	95.0419	93.9083	95.0255	96.7381
		Z	.8625	.8627	.8646	.8628	.8625	.6976
28 Feb 1958	08	X	96.3815	95.1600	94.6696	96.3587	98.0692	97.5127
		Y	98.2522	96.7266	95.0421	93.9085	95.0255	96.7381
		Z	.8627	.8625	.8647	.8628	.8624	.6975
3 Mar 1958	05	X	96.3815	95.1600	94.6696	96.3590	98.0696	97.5131
		Y	98.2523	96.7268	95.0422	93.9085	95.0256	96.7381
		Z	.8626	.8625	.8645	.8627	.8624	.6975
3 Mar 1958	09	X	96.3813	95.1600	94.6697	96.3590	98.0692	97.5126
		Y	98.2523	96.7268	95.0422	93.9087	95.0257	96.7383
		Z	.8625	.8625	.8645	.8626	.8624	.6975
4 Mar 1958	10	X	96.3813	95.1599	94.6697	96.3591	98.0693	97.5126
		Y	98.2523	96.7268	95.0421	93.9086	95.0258	96.7382
		Z	.8627	.8625	.8645	.8628	.8625	.6975
10 Mar 1958	11	X	96.3814	95.1599	94.6696	96.3589	98.0692	97.5126
		Y	98.2523	96.7269	95.0422	93.9089	95.0258	96.7383
		Z	.8625	.8623	.8644	.8626	.8622	.6973
11 Mar 1958	12	X	96.3812	95.1597	94.6696	96.3590	98.0692	97.5125
		Y	98.2524	96.7270	95.0423	93.9086	95.0259	96.7384
		Z	.8627	.8625	.8643	.8626	.8626	.6975
21 Mar 1958	13	X	96.3812	95.1597	94.6694	96.3587	98.0691	97.5124
		Y	98.2524	96.7272	95.0423	93.9086	95.0256	96.7382
		Z	.8621	.8621	.8639	.8619	.8619	.6971
21 Mar 1958	14	X						97.5124
		Y						96.7384
		Z						.6968
28 Mar 1958	15	X	96.3809	95.1597	94.6693	96.3586	98.0691	97.5123
		Y	98.2521	96.7268	95.0420	93.9085	95.0254	96.7381
		Z	.8600	.8602	.8620	.8603	.8600	.6951
2 Apr 1958	16	X	96.3809	95.1597	94.6695	96.3589	98.0691	97.5125
		Y	98.2522	96.7267	95.0421	93.9084	95.0255	96.7380
		Z	.8621	.8623	.8642	.8623	.8638	.6973

TABLE 1 (continued)

XYZ COORDINATES FROM DIFFERENT PIT CALIBRATIONS
(Meters)

Date of Calibration	Cal. No.		Control Point Number 9	15	17	19	23	25
2 Apr 1958	17	X	96.3810	95.1598	94.6697	96.3590	98.0690	97.5124
		Y	98.2522	96.7268	95.0423	93.9089	95.0258	96.7382
		Z	.8621	.8623	.8643	.8627	.8624	.6975
10 Apr 1958	18	X	96.3807	95.1598	94.6693	96.3587	98.0689	97.5122
		Y	98.2522	96.7269	95.0421	93.9085	95.0255	96.7381
		Z	.8629	.8632	.8648	.8633	.8631	.6981
10 Apr 1958	19	X	96.3817	95.1601	94.6699	96.3593	98.0692	97.5126
		Y	98.2522	96.7272	95.0427	93.9087	95.0254	96.7377
		Z	.8625	.8622	.8644	.8627	.8624	.6979
11 Apr 1958	20	X	96.3805	95.1596	94.6693	96.3587	98.0688	97.5121
		Y	98.2521	96.7267	95.0421	93.9088	95.0257	96.7381
		Z	.8629	.8632	.8664	.8635	.8630	.6981
17 Apr 1958	21	X	96.3810	95.1598	94.6696	96.3589	98.0690	97.5123
		Y	98.2522	96.7268	95.0423	93.9090	95.0260	96.7383
		Z	.8631	.8632	.8651	.8634	.8634	.6984
22 Apr 1958	22	X	96.3816	95.1605	94.6703	96.3594	98.0690	97.5124
		Y	98.2519	96.7272	95.0427	93.9088	95.0254	96.7376
		Z	.8622	.8622	.8641	.8623	.8619	.6972
22 Apr 1958	23	X	96.3804	95.1591	94.6689	96.3585	98.0686	97.5119
		Y	98.2517	96.7261	95.0412	93.9079	95.0251	96.7375
		Z	.8624	.8625	.8645	.8629	.8627	.6975
6 May 1958	24	X	96.3822	95.1601	94.6694	96.3594	98.0695	97.5132
		Y	98.2523	96.7277	95.0431	93.9087	95.0249	96.7372
		Z	.8622	.8617	.8633	.8621	.8623	.6974
6 May 1958	25	X	96.3804	95.1595	94.6693	96.3585	98.0687	97.5120
		Y	98.2524	96.7273	95.0430	93.9089	95.0256	96.7382
		Z	.8610	.8618	.8640	.8620	.8614	.6965
7 May 1958	26	X	96.3812	95.1597	94.6694	96.3590	98.0690	97.5122
		Y	98.2522	96.7268	95.0422	93.9088	95.0258	96.7380
		Z	.8616	.8617	.8637	.8621	.8619	.6968
8 May 1958	27	X	96.3808	95.1595	94.6692	96.3586	98.0687	97.5120
		Y	98.2522	96.7268	95.0422	93.9089	95.0257	96.7380
		Z	.8611	.8614	.8634	.8617	.8615	.6964

TABLE 1 (continued)

XYZ COORDINATES FROM DIFFERENT PIT CALIBRATIONS
(Meters)

Date of Calibration	Cal. No.		Control Point Number 9	15	17	19	23	25
15 May 1958	28	X	96.3810	95.1596	94.6695	96.3589	98.0689	97.5123
		Y	98.2523	96.7269	95.0426	93.9090	95.0259	96.7382
		Z	.8617	.8616	.8638	.8620	.8622	.6972
21 May 1958	29	X	96.3812	95.1595	94.6694	96.3587	98.0690	97.5124
		Y	98.2521	96.7268	95.0420	93.9084	95.0252	96.7379
		Z	.8616	.8614	.8635	.8616	.8627	.6966
22 May 1958	30	X	96.3808	95.1595	94.6693	96.3588	98.0686	97.5120
		Y	98.2519	96.7266	95.0422	93.9085	95.0256	96.7378
		Z	.8610	.8610	.8631	.8612	.8614	.6962
23 May 1958	31	X	96.3807	95.1596	94.6692	96.3587	98.0686	97.5121
		Y	98.2524	96.7270	95.0426	93.9093	95.0261	96.7383
		Z	.8612	.8613	.8635	.8620	.8619	.6968
26 May 1958	32	X	96.3809	95.1597	94.6695	96.3589	98.0689	97.5124
		Y	98.2521	96.7266	95.0421	93.9086	95.0257	96.7380
		Z	.8612	.8612	.8633	.8614	.8616	.6968
27 May 1958	33	X	96.3807	95.1595	94.6693	96.3587	98.0686	97.5120
		Y	98.2520	96.7265	95.0420	93.9085	95.0257	96.7378
		Z	.8624	.8622	.8647	.8634	.8635	.6981
2 Jun 1958	34	X	96.3809	95.1594	94.6691	96.3585	98.0685	97.5120
		Y	98.2520	96.7267	95.0421	93.9089	95.0256	96.7379
		Z	.8629	.8624	.8644	.8630	.8631	.6980
4 Jun 1958	35	X	96.3812	95.1597	94.6695	96.3600	98.0688	97.5123
		Y	98.2519	96.7266	95.0421	93.9067	95.0256	96.7377
		Z	.8624	.8623	.8642	.8622	.8628	.6978
5 Jun 1958	36	X	96.3816	95.1604	94.6699	96.3592	98.0690	97.5125
		Y	98.2520	96.7272	95.0427	93.9089	95.0257	96.7378
		Z	.8622	.8621	.8643	.8624	.8624	.6976
5 Jun 1958	37	X	96.3804	95.1591	94.6689	96.3582	98.0683	97.5116
		Y	98.2525	96.7271	95.0427	93.9089	95.0258	96.7381
		Z	.8614	.8615	.8640	.8618	.8616	.6968

TABLE 2

QUADRILATERAL ANGLES
SHOWING ADJUSTED ANGLES AND CORRECTIONS TO OBSERVED ANGLES

Date	Cal. No.	Angle 1	Angle 2	Angle 3	Angle 4	Angle 5	Angle 6	Angle 7	Angle 8	Mean Error Single Angle
		42° 14'	47° 31'	47° 39'	42° 26'	42° 21'	47° 24'	47° 46'	42° 33'	
9 Feb 1958	01	51".27	56".68	48".84	64".16	10".32	69".98	35".54	23".21	1".62
		-.33	-1.52	.04	-.84	1.02	-1.22	.34	-2.19	
3 Mar 1958	05	46.56	65.85	40.91	57.23	16.01	64.70	42.06	26.68	1.06
		-.84	-.35	-.79	-.57	-1.09	-.60	-1.04	-.32	
10 Apr 1958	19	50.88	68.51	33.87	55.24	22.38	65.26	37.12	26.74	2.66
		.88	1.51	.37	-1.76	-3.12	-1.24	-2.38	2.24	
22 Apr 1958	22	46.71	74.22	30.04	47.28	28.46	65.97	38.29	29.03	.90
		-.79	-.28	-.96	-.22	-1.04	-.03	-.71	.03	
6 May 1958	24	51.45	70.87	27.51	55.67	25.95	59.12	39.26	30.17	3.98
		.95	.37	-1.49	-3.33	-5.55	-.88	-2.74	3.17	
5 Jun 1958	36	49.15	75.78	26.09	48.48	29.65	63.53	38.34	28.98	3.11
		.15	-2.22	-2.91	-3.02	-3.85	.03	-.66	.98	

TABLE 3

ADJUSTED COORDINATES OF QUADRILATERAL STATIONS

Date	Cal. No.	Sta. A	Sta. B	Sta. C	Sta. D(enforced)
9 Feb 1958	01	X 92.7745	X 92.7614	X 100.0000	X 100.0000
		Y 100.0420	Y 93.4311	Y 93.4023	Y 100.0000
3 Mar 1958	05	X 92.7745	X 92.7610	X 100.0000	X 100.0000
		Y 100.0423	Y 93.4310	Y 93.4023	Y 100.0000
10 Apr 1958	19	X 92.7746	X 92.7610	X 100.0000	X 100.0000
		Y 100.0422	Y 93.4308	Y 93.4023	Y 100.0000
22 Apr 1958	22	X 92.7744	X 92.7607	X 100.0000	X 100.0000
		Y 100.0423	Y 93.4306	Y 93.4023	Y 100.0000
6 May 1958	24	X 92.7748	X 92.7610	X 100.0000	X 100.0000
		Y 100.0424	Y 93.4309	Y 93.4023	Y 100.0000
5 Jun 1958	36	X 92.7746	X 92.7607	X 100.0000	X 100.0000
		Y 100.0423	Y 93.4306	Y 93.4023	Y 100.0000

TABLE 4

ADJUSTED LENGTHS BETWEEN QUADRILATERAL STATIONS
(Meters)

Date	Cal. No.	A-B	B-C	C-D*	D-A	A-C	B-D
9 Feb 1958	01	6.61094	7.23862	6.59770	7.22565	9.81294	9.77485
3 Mar 1958	05	6.61130	7.23910	6.59770	7.22566	9.81318	9.77521
10 Apr 1958	19	6.61137	7.23902	6.59770	7.22552	9.81296	9.77531
22 Apr 1958	22	6.61173	7.23936	6.59770	7.22570	9.81318	9.77573
6 May 1958	24	6.61148	7.23907	6.59770	7.22530	9.81293	9.77529
5 Jun 1958	36	6.61169	7.23931	6.59770	7.22553	9.81305	9.77567

*C-D enforced

TABLE 5

ADJUSTED Z COORDINATES
(Meters)

Date of Calibration	Cal. No.	9	15	17	19	23	25
28 Feb 1958	07	.8625	.8627	.8646	.8628	.8625	.6976
28 Feb 1958	08	.8627	.8625	.8647	.8628	.8624	.6975
3 Mar 1958	05	.8626	.8625	.8645	.8627	.8624	.6975
3 Mar 1958	09	.8625	.8625	.8645	.8626	.8624	.6975
4 Mar 1958	10	.8627	.8625	.8645	.8628	.8625	.6975
10 Mar 1958	11	.8625	.8623	.8644	.8626	.8622	.6973
11 Mar 1958	12	.8627	.8625	.8643	.8626	.8626	.6975
21 Mar 1958	13	.8621	.8621	.8639	.8619	.8619	.6971
21 Mar 1958	14						.6968
28 Mar 1958	15	.8600	.8602	.8620	.8603	.8600	.6951
2 Apr 1958	16	.8621	.8623	.8642	.8623	.8638	.6973
7 Apr 1958	17	.8621	.8623	.8643	.8627	.8624	.6975
10 Apr 1958	18	.8629	.8632	.8648	.8633	.8631	.6981
10 Apr 1958	19	.8625	.8622	.8644	.8627	.8624	.6979
11 Apr 1958	20	.8629	.8632	.8664	.8635	.8630	.6981
17 Apr 1958	21	.8631	.8632	.8651	.8634	.8634	.6984
22 Apr 1958	22	.8622	.8622	.8641	.8623	.8619	.6972
22 Apr 1958	23	.8624	.8625	.8645	.8629	.8627	.6975
6 May 1958	24	.8622	.8617	.8633	.8621	.8623	.6974
6 May 1958	25	.8610	.8618	.8640	.8620	.8614	.6965
7 May 1958	26	.8616	.8617	.8637	.8621	.8619	.6968
8 May 1958	27	.8611	.8614	.8634	.8617	.8615	.6964
15 May 1958	28	.8617	.8616	.8638	.8620	.8622	.6972

TABLE 5 (continued)

ADJUSTED Z COORDINATES
(Meters)

Date of Calibration	Cal. No.	9	15	17	19	23	25
21 May 1958	29	.8616	.8614	.8635	.8616	.8627	.6966
22 May 1958	30	.8610	.8610	.8631	.8612	.8614	.6962
23 May 1958	31	.8612	.8613	.8635	.8620	.8619	.6968
26 May 1958	32	.8612	.8612	.8633	.8614	.8616	.6968
27 May 1958	33	.8624	.8622	.8647	.8634	.8635	.6981
2 Jun 1958	34	.8629	.8624	.8644	.8630	.8631	.6980
4 Jun 1958	35	.8624	.8623	.8642	.8622	.8628	.6978
5 Jun 1958	36	.8622	.8621	.8643	.8624	.8624	.6976
5 Jun 1958	37	.8614	.8615	.8640	.8618	.8616	.6968

TABLE 6

COMPUTED DISTANCE BETWEEN CONTROL POINTS ON GUN ROD
(11 Feb to 21 May Runs)

Date	Cal. No.	Pt. 26 X Y Z	Pt. 27 X Y Z	ΔX ΔY ΔZ	Dist. Meters	v mm	vv
11 Feb 1958	02	97.3123	97.3258	.0135			
		96.6148	96.6174	.0026			
		5.0124	4.1603	-.8521	.8522	.04	.0016
13 Feb 1958	03	97.3373	97.3545	.0172			
		96.6000	96.6023	.0023			
		5.0304	4.1784	-.8520	.8522	.04	.0016
20 Feb 1958	04	97.3464	97.3636	.0172			
		96.7039	96.7069	.0030			
		5.0387	4.1868	-.8519	.8521	.14	.0196
20 Feb 1958	06	97.3464	97.3636	.0172			
		96.7039	96.7069	.0030			
		5.0387	4.1868	-.8519	.8521	.14	.0196
28 Feb 1958	07	97.3212	97.3370	.0158			
		96.7851	96.7890	.0039			
		5.0213	4.1692	-.8521	.8522	.04	.0016
28 Feb 1958	08	97.3855	97.4021	.0166			
		96.7554	96.7588	.0034			
		5.0282	4.1762	-.8520	.8522	.04	.0016
3 Mar 1958	09	97.3370	97.3535	.0165			
		96.7143	96.7185	.0042			
		5.0326	4.1809	-.8517	.8519	.34	.1156
4 Mar 1958	10	97.3668	97.3844	.0176			
		96.6877	96.6913	.0036			
		5.0431	4.1911	-.8520	.8522	.04	.0016
10 Mar 1958	11	97.3689	97.3838	.0149			
		96.7415	96.7433	.0018			
		5.0250	4.1729	-.8521	.8522	.04	.0016
11 Mar 1958	12	97.3446	97.3614	.0168			
		96.7514	96.7530	.0016			
		5.0444	4.1924	-.8520	.8522	.04	.0016

TABLE 6 (continued)

COMPUTED DISTANCE BETWEEN CONTROL POINTS ON GUN ROD
(11 Feb to 21 May Runs)

Date	Cal. No.	Pt. 26 X Y Z	Pt. 27 X Y Z	ΔX ΔY ΔZ	Dist. Meters	v mm	vv
21 Mar 1958	13	97.3659 96.6885 5.0372	97.3837 96.6878 4.1851	.0178 -.0007 -.8521	.8523	-.06	.0036
21 Mar 1958 (recalibration)	14	97.3651 96.6994 5.0356	97.3825 96.6987 4.1834	.0174 -.0007 -.8522	.8524	-.16	.0256
28 Mar 1958	15	97.3725 96.6807 5.0438	97.3890 96.6844 4.1917	.0165 .0037 -.8521	.8523	-.06	.0036
2 Apr 1958	16	97.3574 96.7266 5.0420	97.3738 96.7314 4.1900	.0164 .0048 -.8520	.8522	.04	.0016
7 Apr 1958	17	97.3702 96.6695 5.0416	97.3875 96.6740 4.1895	.0173 .0045 -.8521	.8523	-.06	.0036
10 Apr 1958	18	97.3819 96.7150 5.0381	97.3973 96.7190 4.1858	.0154 .0040 -.8523	.8524	-.16	.0256
11 Apr 1958	20	97.3627 96.7056 5.0429	97.3983 96.6954 4.1908	.0356 -.0102 -.8521	.8529	-.66	.4356
17 Apr 1958	21	97.3710 96.6874 5.0359	97.3867 96.6902 4.1837	.0157 .0028 -.8522	.8523	-.06	.0036
22 Apr 1958	23	97.3508 96.6783 5.0296	97.3657 96.6808 4.1775	.0149 .0025 -.8521	.8522	.04	.0016
6 May 1958	25	97.3693 96.7135 5.0147	97.3832 96.7179 4.1627	.0139 .0044 -.8520	.8521	.14	.0196
7 May 1958	26	97.3642 96.6982 5.0360	97.3795 96.7016 4.1840	.0153 .0034 -.8520	.8521	.14	.0196

TABLE 6 (continued)

COMPUTED DISTANCE BETWEEN CONTROL POINTS ON GUN ROD
(11 Feb to 21 May Runs)

Date	Cal. No.	Pt. 26 X / Y / Z	Pt. 27 X / Y / Z	ΔX / ΔY / ΔZ	Dist. Meters	v mm	vv
8 May 1958	27	97.3414	97.3560	.0146			
		96.7118	96.7140	.0022			
		5.0270	4.1748	-.8522	.8523	-.06	.0036
15 May 1958	28	97.3828	97.3982	.0154			
		96.7827	96.7852	.0025			
		5.0304	4.1783	-.8521	.8522	.04	.0016
21 May 1958	29	97.3548	97.3699	.0151			
		96.6718	96.6730	.0012			
		5.0298	4.1776	-.8522	.8523	-.06	.0036

Average = .85224 m.e. = ± 0.18mm

COMPUTED DISTANCE BETWEEN CONTROL POINTS ON GUN ROD
(22 May to 5 June Runs)

Date	Cal. No.	Pt. 26 X / Y / Z	Pt. 27 X / Y / Z	ΔX / ΔY / ΔZ	Dist. Meters	v mm	vv
22 May 1958	30	97.3570	97.3716	.0146			
		96.6485	96.6497	.0012			
		5.0245	4.1735	-.8510	.8511	-3.07	9.4249
23 May 1958	31	97.3693	97.3835	.0142			
		96.7041	96.7066	.0025			
		5.0218	4.1766	-.8452	.8453	2.73	7.4529
26 May 1958	32	97.3521	97.3667	.0146			
		96.6834	96.6860	.0026			
		5.0187	4.1759	-.8428	.8427	5.13	26.3169
27 May 1958	33	97.3674	97.3816	.0142			
		96.7330	96.7359	.0029			
		5.0304	4.1788	-.8516	.8517	-3.67	13.4689

TABLE 6 (continued)

COMPUTED DISTANCE BETWEEN CONTROL POINTS ON GUN ROD
(22 May to 5 June Runs)

Date	Cal. No.	Pt. 26 X Y Z	Pt. 27 X Y Z	ΔX ΔY ΔZ	Dist. Meters	v mm	vv
2 Jun 1958	34	97.3725 96.6409 5.0332	97.3891 96.6455 4.1823	.0166 .0046 -.8509	.8511	-3.07	9.4249
4 Jun 1958	35	97.3612 96.6928 5.0329	97.3770 96.6956 4.1898	.0158 .0028 -.8431	.8432	4.83	23.3289
5 Jun 1958	37	97.3582 96.7313 5.0328	97.3728 96.7353 4.1820	.0146 .0040 -.8508	.8509	-2.87	8.2369

Average = .84803 m.e. = ± 4.03mm

TABLE 7

MEAN ERRORS OF CONTROL POINTS
FROM 3 MARCH PIT CALIBRATION

Control Point	m.e. single direction (sec's of arc)	m.e. ΔX (mm)	m.e. ΔY (mm)
7	± 7.13	$\pm .1951$	$\pm .0868$
8	± 1.43	$\pm .0297$	$\pm .0135$
9	± 2.76	$\pm .0551$	$\pm .0387$
19	± 8.63	$\pm .2335$	$\pm .1054$
20	± 8.38	$\pm .1730$	$\pm .1148$
21	± 8.06	$\pm .1479$	$\pm .1228$
Average	± 6.06	$\pm .1390$	$\pm .0803$

Approximate uncertainty in control point positions: =

$$\sqrt{\overline{.1390}^{2} + \overline{.0803}^{2}} = .1605\text{mm}$$

TABLE 8

MEAN ERRORS OF CONTROL POINTS
FROM 22 APRIL PIT CALIBRATION

Control Point	m.e. single direction (sec's of arc)	m.e. ΔX (mm)	m.e. ΔY (mm)
7	± 12.73	$\pm .3481$	$\pm .1550$
8	± 20.32	$\pm .4210$	$\pm .2774$
9	± 18.53	$\pm .3697$	$\pm .2600$
19	± 36.49	$\pm .9872$	$\pm .4460$
20	± 33.71	$\pm .6959$	$\pm .4618$
21	± 31.32	$\pm .5745$	$\pm .4771$
Average	± 25.52	$\pm .5661$	$\pm .3462$

Approximate uncertainty in control point positions: =

$$\sqrt{\overline{.5661}^{\,2} + \overline{.3462}^{\,2}} = .6636\text{mm}$$

TABLE 9

MEAN ERRORS OF CONTROL POINTS
FROM 6 MAY PIT CALIBRATION

Control Point	m.e. single direction (sec's of arc)	m.e. ΔX (mm)	m.e. ΔY (mm)
7	± 26.52	$\pm .7250$	$\pm .3227$
8	± 34.08	$\pm .7059$	$\pm .4651$
9	± 34.52	$\pm .6888$	$\pm .4844$
19	± 47.30	± 1.2795	$\pm .5778$
20	± 45.47	$\pm .9389$	$\pm .6230$
21	± 44.66	$\pm .8193$	$\pm .6803$
Average	± 38.76	$\pm .8596$	$\pm .5256$

Approximate uncertainty in control point positions: =

$$\sqrt{\overline{.8596}^{2} + \overline{.5256}^{2}} = 1.0076\text{mm}$$

TABLE 10

RESULTS OF TWO METHODS OF REDUCING
PIT CONTROL POINT COORDINATES

Pt. No.	I* X	I* Y	II* X	II* Y	III* ΔX	III* ΔY
	METERS					
	3 March Quadrilateral 1".06					
7	96.40427	96.57193	96.40434	99.57197	-.00007	-.00004
8	96.38496	98.45211	96.38493	98.45212	.00003	-.00001
9	96.38155	98.25234	96.38151	98.25234	.00004	.00000
19	96.35897	93.90848	96.35900	93.90853	-.00003	-.00005
20	96.37503	95.00592	96.37497	95.00590	.00006	.00002
21	96.39222	95.81815	96.39208	95.81801	.00014	.00014
					Average =	.00005
	22 April Quadrilateral 0".90					
7	96.40391	99.57160	96.40386	99.57170	.00005	-.00010
8	96.38472	98.45165	96.38498	98.45162	-.00026	.00003
9	96.38129	98.25198	96.38157	98.25192	-.00028	.00006
19	96.35900	93.90888	96.35938	93.90881	-.00038	.00007
20	96.37502	95.00617	96.37448	95.00619	.00054	-.00002
21	96.39207	95.81824	96.39117	95.81852	.00090	-.00028
					Average =	.00025
	6 May Quadrilateral 3".98					
7	96.40452	99.57210	96.40414	99.57208	.00038	.00002
8	96.38502	98.45214	96.38555	98.45218	-.00053	-.00004
9	96.38155	98.25230	96.38219	98.25234	-.00064	-.00004
19	96.35886	93.90869	96.35938	93.90873	-.00052	-.00004
20	96.37520	95.00597	96.37444	95.00597	+.00076	.00000
21	96.39231	95.81814	96.39090	95.81803	.00141	.00011
					Average =	.00037

* I - Final XY coordinates of Control Points by Least Squares Adjustment

II - Final XY coordinates of Control Points by Averaging the 6 sets of Coordinates as obtained from the 6 Baselines.

III - Difference between the 2 methods.

TABLE 11

COMPARISONS OF TWO SETS OF RC-7 CAMERA ORIENTATIONS OVER CALIBRATION PIT
(enforcing c, x_p and y_p)

Cal. No.	Date Control Pts. Surveyed	Date of Mission	m.e.*	α / m.e. α	ω / m.e. ω	κ / m.e. κ	X_o / m.e. X_o	Y_o / m.e. Y_o	Z_o / m.e. Z_o
				←	grads	→	←	meters	→
	SET 1								
05	3 March	7 May	4.9	198.226	.224	-49.480	96.437	96.705	5.668
				.0065	.0052	.0018	.0005	.0004	.0010
24	6 May	7 May	8.7	198.215	.235	-49.465	96.436	96.704	5.668
				.0115	.0092	.0032	.0009	.0008	.0017
	SET 2								
05	3 March	8 May	5.9	198.277	.206	-49.395	96.413	96.720	5.661
				.0077	.0063	.0021	.0006	.0005	.0012
24	6 May	8 May	8.4	198.226	.216	-49.380	96.412	96.720	5.661
				.0110	.0089	.0030	.0009	.0008	.0016

* Mean error in microns of a single plate measurement of unit weight

TABLE 12

COMPARATOR CODE

(RC-7 Camera over the range for 21 March M2/P1)

	INPUT*		OUTPUT**			
	^{1}x	^{1}y	^{1}x		^{1}y	
	(meters)		(meters)			
Fiducial Mark						
100	.128831	.115985				
200	.259609	.245193				
300	.258816	.115225				
400	.129619	.245968				
Control Point						
9	.253860	.225482	.73968716	-01	-.99804294	-02
24	.146768	.141619	-.61147444	-01	.56284817	-02
21	.158575	.238320	.15213875	-01	.66116460	-01
12	.240821	.130847	-.18129943	-02	-.68133299	-01
13	.223125	.230030	.55307777	-01	.14855680	-01
16	.211016	.133103	-.21431156	-01	-.45587507	-01
17	.192655	.235042	.37155506	-01	.39837333	-01

* Original plate measurements as given by comparator; multiple settings were made, however only averages are shown here.

** Reduced or final plate measurements; these values are fed into the camera orientation code.

TABLE 13

RC-7 ORIENTATION OVER THE CALIBRATION PIT
(enforcing c, x_p & y_p)

Showing residuals in the plate measurements and control points

Calibration Date 21 March

Control Point No.	Residual in the Plate Measurement (microns)		Residual in the Control Points (millimeters)		
	ℓ_x	ℓ_y	X	Y	Z
1	-2.1	-2.5	-.014	.159	-.008
2	0.7	-2.3	-.100	.054	-.034
3	-1.4	-1.2	.008	.088	.003
4	1.3	-0.2	-.051	-.038	-.050
5	-0.5	0.4	.028	.003	.011
6	0.6	-0.2	-.025	-.012	-.009
7	0.6	2.2	.054	-.095	-.057
8	-0.4	1.5	.065	-.036	-.014
9	1.6	0.6	-.035	-.075	-.024
10	-0.2	-2.7	-.086	.099	.106
11	-1.3	0.8	.071	.017	-.019
12	-0.2	1.8	.067	-.054	-.025
13	0.8	-2.1	-.099	.042	.060
14	-0.3	0.7	.033	-.013	-.012
15	1.7	-0.4	-.072	-.044	.019
16	-3.3	0.3	.123	.099	-.125
17	1.7	2.1	.009	-.126	.040
18	0.8	0.4	-.014	-.038	.008
19	-0.1	-2.7	-.085	.095	-.054
20	2.1	0.0	-.074	-.070	.026
21	-0.2	-0.5	-.011	.022	-.004
22	1.7	1.9	.002	-.125	.071
23	0.3	2.9	.087	-.110	.068
24	-0.5	0.3	.029	.005	.002
25	-3.7	-0.9	.099	.163	.022

TABLE 14

RC-7 ORIENTATION OVER THE RANGE
(enforcing X_o, Y_o, Z_o)

Showing residuals in the plate measurements and control points

Date: 21 Mar M2/P1

Control Point No.	Residual in the Plate Measurement (microns)		Residual in the Control Points (meters)		
	ℓ_x	ℓ_y	X	Y	Z
9	-1.2	1.2	.16	.02	.10
12	-2.7	-1.5	.05	.29	.09
13	-0.3	-1.0	-.06	.08	.03
16	2.3	-1.1	-.22	-.11	-.01
17	0.4	0.8	.03	-.08	-.04
21	-0.3	1.3	.12	-.05	-.07
24	1.7	0.3	-.08	-.14	.09

TABLE 15

CAMERA ORIENTATION RESULTS OF RC-7 OVER RANGE

(enforcing c, x_p & y_p**)

Miss. No.	Date	Miss./Pass/Rd.	Points***	m.e.****		METERS			GRADS	
					X_o m.e. X_o	Y_o m.e. Y_o	Z_o m.e. Z_o	α_g m.e. α_g	ω_g m.e. ω_g	κ_g m.e. κ_g
1	21 Mar	1/2	4	1.1	1851.960	-613.408	9599.050	197.820	- .160	-53.159
					.568	.615	.150	.0034	.0033	.0009
2	21 Mar	1/2*	4	1.5	1853.062	-613.545	9599.283	197.821	- .161	-53.156
					.779	.843	.206	.0047	.0046	.0012
3	21 Mar	2/1	7	2.4	2055.657	-495.023	9482.221	197.755	.279	-57.310
					.729	.866	.240	.0043	.0047	.0014
4	28 Mar	2/2	6	2.8	1766.157	-204.972	9589.176	196.129	2.246	-55.583
					.936	1.061	.312	.0054	.0057	.0017
5	10 Apr	3/1-1	4	1.0	1779.470	-687.188	9660.671	197.532	-1.933	-53.280
					.515	.559	.138	.0031	.0030	.0008
6	10 Apr	3/1-1*	5	1.6	1780.491	-690.363	9660.614	197.536	-1.918	-53.279
					.774	.874	.206	.0047	.0047	.0012
7	10 Apr	3/1-2	6	2.4	3546.846	-778.052	9663.365	197.916	.660	-52.597
					.820	.914	.271	.0046	.0049	.0016
8	10 Apr	3/1-2*	6	1.1	3548.360	-777.666	9663.799	197.918	.657	-52.601
					.368	.410	.122	.0021	.0022	.0007
9	11 Apr	1/2-1	6	2.6	2240.064	-273.456	9546.289	197.641	2.951	-53.364
					.840	.956	.264	.0049	.0052	.0015
10	11 Apr	1/2-2	6	1.5	3608.286	-351.200	9561.168	197.708	1.243	-52.739
					.476	.568	.152	.0028	.0031	.0009
11	11 Apr	1/3-1	7	2.2	1819.064	-549.332	9639.000	197.836	.396	-53.886
					.686	.814	.232	.0040	.0044	.0013

TABLE 15 (continued)

CAMERA ORIENTATION RESULTS OF RC-7 OVER RANGE
(enforcing c, x_p & y_p**)

Miss. No.	Date	Miss./Pass/Rd.	Points***	m.e.****	METERS X_o / m.e. X_o	Y_o / m.e. Y_o	Z_o / m.e. Z_o	GRADS α_g / m.e. α_g	ω_g / m.e. ω_g	κ_g / m.e. κ_g
12	11 Apr	1/3-2	7	1.8	3288.796	-619.851	9635.572	198.587	- .115	-53.407
					.557	.671	.177	.0032	.0036	.0010
13	7 May	1/1	5	1.8	1957.487	-488.366	9686.621	197.257	-2.991	-56.811
					.696	.869	.228	.0040	.0048	.0012
14	7 May	1/2	5	2.7	1747.189	-212.970	9533.794	196.997	1.714	-57.526
					1.028	1.242	.314	.0061	.0068	.0018
15	7 May	1/4	5	2.9	1791.185	-390.422	9617.879	196.726	.734	-58.140
					1.121	1.374	.353	.0066	.0075	.0020
16	8 May	3/1	6	3.4	1808.929	-448.490	9699.414	197.091	2.771	-50.466
					1.285	1.522	.448	.0076	.0080	.0023
17	8 May	3/2	4	2.8	1829.620	-645.439	9685.137	197.640	1.313	-51.552
					1.446	1.556	.379	.0087	.0084	.0022
			Avg. m.e. =	2.1	.801	.924	.246	.0047	.0050	.0014

* Check Run

** $c = .10055$, $x_p = y_p = 0$

*** number of control points measured

**** mean error in microns of a single measurement of unit weight

TABLE 16

CAMERA ORIENTATION RESULTS OF RC-7 OVER RANGE
(enforcing X_o Y_o Z_o from Askania Results)

					METERS			GRADS			c, x_p, y_p in METERS; m.e. in microns		
	Date	Miss./Pass/Rd.	Points**	m.e.***	X_o	Y_o	Z_o	α_g / m.e. α_g	ω_g / m.e. ω_g	κ_g / m.e. κ_g	c / m.e. c	x_p / m.e. x_p	y_p / m.e. y_p
1	21 Mar	1/2	4	.2	1838.414	-614.452	9602.599	197.819	- .159	-53.159	.100594	-.000100	.000097
								.0005	.0005	.0001	.3	1.0	1.0
2	21 Mar	2/1	7	2.6	2047.767	-499.042	9486.338	197.754	.280	-57.309	.100598	-.000082	.000038
								.0047	.0052	.0015	2.7	9.2	9.4
3	21 Mar	1/2*	4	2.2	1838.358	-614.480	9602.246	197.819	- .161	-53.156	.100588	-.000108	.000106
								.0067	.0066	.0017	3.2	12.4	12.1
4	28 Mar	2/2	6	3.5	1761.247	-206.600	9593.317	196.125	2.246	-55.583	.100599	-.000039	.000023
								.0066	.0071	.0021	3.7	13.2	12.8
5	10 Apr	3/1-1	4	.9	1773.690	-688.397	9664.376	197.530	-1.933	-53.278	.100591	-.000048	.000033
								.0028	.0027	.0007	1.5	5.1	4.9
6	10 Apr	3/1-2	6	2.4	3544.589	-782.672	9668.232	197.914	.660	-52.596	.100602	-.000048	-.000018
								.0045	.0048	.0015	2.8	8.9	8.8
7	10 Apr	3/1-1*	5	.9	1773.823	-688.470	9664.254	197.536	-1.919	-53.278	.100592	-.000031	.000062
								.0028	.0028	.0007	1.5	5.2	5.0
8	10 Apr	3/1-2*	6	1.4	3544.336	-782.942	9667.929	197.916	.658	-52.600	.100596	-.000065	-.000082
								.0026	.0028	.0009	1.7	5.2	5.2
9	11 Apr	1/2-1	6	3.0	2236.449	-275.771	9550.742	197.638	2.952	-53.364	.100601	-.000037	.000010
								.0056	.0061	.0018	3.2	11.2	10.9

TABLE 16 (continued)

CAMERA ORIENTATION RESULTS OF RC-7 OVER RANGE
(enforcing X_o Y_o Z_o from Askania Results)

	Date	Miss./Pass/Rd.	Points**	m.e.***	METERS X_o	Y_o	Z_o	GRADS α_g / m.e. α_g	ω_g / m.e. ω_g	κ_g / m.e. κ_g	c, x_p, y_p in METERS; m.e. in microns — c / m.e. c	x_p / m.e. x_p	y_p / m.e. y_p
10	11 Apr	1/2-2	6	1.7	3600.896	-357.400	9566.298	197.706	1.244	-52.739	.100610	-.000096	.000010
								.0030	.0035	.0010	1.9	6.2	6.2
11	11 Apr	1/3-1	7	2.1	1815.220	-552.959	9643.207	197.836	.396	-53.886	.100596	-.000053	.000004
								.0040	.0043	.0013	2.3	7.6	8.0
12	11 Apr	1/3-2	7	2.0	3283.635	-626.962	9640.157	198.586	- .114	-53.406	.100599	-.000089	-.000011
								.0036	.0041	.0011	2.1	7.1	7.4
13	7 May	1/1	5	2.0	1953.168	-488.474	9692.633	197.255	-2.993	-56.810	.100615	-.000029	.000026
								.0042	.0051	.0013	2.9	8.6	8.8
14	7 May	1/2	5	3.1	1744.571	-217.855	9539.128	196.995	1.715	-57.526	.100611	-.000051	-.000012
								.0070	.0077	.0021	3.6	13.1	14.3
15	7 May	1/4	5	3.5	1786.280	-395.136	9622.045	196.724	.734	-58.140	.100598	-.000067	.000007
								.0080	.0090	.0024	4.3	15.3	16.2
16	8 May	3/1	6	3.2	1803.757	-461.058	9702.347	197.085	2.766	-50.465	.100596	-.000127	-.000066
								.0071	.0081	.0023	4.1	13.7	14.6
17	8 May	3/2	4	2.4	1826.179	-646.738	9690.270	197.640	1.314	-51.553	.100606	-.000031	.000017
								.0074	.0072	.0019	3.4	13.6	13.4
		Avg. m.e. = 2.2						.0048	.0052	.0014	2.6	9.2	9.4

* Check Run
** Number of control points measured
*** m.e. in microns of a single measurement of unit weight

TABLE 17

CAMERA ORIENTATION RESULTS OF RC-7 OVER CALIBRATION PIT
(enforcing c, x_p & y_p**)

Date	Points***	m.e.****	METERS X_o / m.e. X_o	Y_o / m.e. Y_o	Z_o / m.e. Z_o	GRADS α_g / m.e. α_g	ω_g / m.e. ω_g	κ_g / m.e. κ_g
21 Mar	25	2.6	96.435	96.710	5.666	198.051	- .045	-49.439
			.0003	.0002	.0005	.0034	.0028	.0009
21 Mar*	23	2.4	96.435	96.710	5.665	198.054	- .039	-49.442
			.0004	.0003	.0006	.0047	.0032	.0010
28 May	24	2.7	96.443	96.691	5.677	198.129	.279	-49.318
			.0003	.0003	.0005	.0036	.0035	.0010
10 Apr	22	6.3	96.452	96.719	5.668	198.202	.311	-49.683
			.0011	.0007	.0017	.0138	.0085	.0027
10 Apr*	22	7.1	96.452	96.719	5.667	198.205	.312	-49.685
			.0013	.0008	.0019	.0155	.0096	.0031
11 Apr	22	6.3	96.453	96.701	5.672	198.178	.324	-49.298
			.0009	.0007	.0014	.0114	.0084	.0026
7 May	25	4.9	96.437	96.705	5.668	198.226	.224	-49.480
			.0005	.0004	.0010	.0065	.0052	.0018
8 May	25	5.9	96.413	96.720	5.661	198.277	.206	-49.395
			.0006	.0005	.0012	.0077	.0063	.0021
Avg. m. e. =		4.8	.0007	.0005	.0011	.0083	.0059	.0019

* CHECK RUN

** c = .10055, $x_p = y_p = 0$

*** number of control points measured

**** mean error in microns of a single measurement of unit weight

TABLE 18

COMPARISONS BETWEEN ASKANIA AND RC-7 CAMERA SYSTEMS
(enforcing c, x_p & y_p in RC-7 camera orientations)

Askania minus RC-7 Values (meters)

Miss No.	Miss/Pass Round	(1) ΔX	(2) ΔY	(3) ΔZ before refraction	(4) Askania refraction	(5) Average RC-7 refraction	(6) ΔZ Final value
1	21 Mar 1/2	-13.54	- 1.04	4.55	-1.40	1.00	2.15
2	21 Mar 2/1	- 7.89	- 4.02	5.12	-1.40	"	2.72
3	21 Mar 1/2*	-14.70	- .93	3.96	-1.40	"	1.56
4	28 Mar 2/2	- 4.91	- 1.63	5.14	-1.39	"	2.75
5	10 Apr 3/1-1	- 5.78	- 1.21	4.70	-1.41	"	2.29
6	10 Apr 3/1-2	- 2.26	- 4.62	5.87	-1.57	"	3.30
7	10 Apr 3/1-1*	- 6.67	+ 1.89	4.64	-1.41	"	2.23
8	10 Apr 3/1-2*	- 4.02	- 5.28	5.13	-1.57	"	2.56
9	11 Apr 1/2-1	- 3.62	- 2.32	5.45	-1.39	"	3.06
10	11 Apr 1/2-2	- 7.39	- 6.20	6.13	-1.55	"	3.58
11	11 Apr 1/3-1	- 3.84	- 3.63	5.21	-1.41	"	2.80
12	11 Apr 1/3-2	- 5.16	- 7.11	5.58	-1.53	"	3.05
13	7 May 1/1	- 4.32	- .11	7.01	-1.42	"	4.59
14	7 May 1/2	- 2.62	- 4.88	6.33	-1.38	"	3.95
15	7 May 1/4	- 4.90	- 4.71	5.16	-1.40	"	2.76
16	8 May 3/1	- 5.17	-12.57	3.93	-1.41	"	1.52
17	8 May 3/2	- 3.44	- 1.30	6.13	-1.42	"	3.71

*Check runs

TABLE 19

REFRACTION CORRECTIONS FROM ASKANIA CAMERAS TO RC-7 CAMERA

Miss. No.	Mission		Refraction Corrections (meters) Pi C to RC-7 (1)	Pi D to RC-7 (2)	Difference (3)	Mean (4)
1	21 Mar	1/2	-1.33	-1.47	.14	-1.40
2	21 Mar	2/1	-1.34	-1.45	.11	-1.40
3	21 Mar	1/2*	-1.33	-1.47	.14	-1.40
4	28 Mar	2/2	-1.37	-1.41	.04	-1.39
5	10 Apr	3/1-1	-1.33	-1.49	.16	-1.41
6	10 Apr	3/1-2	-1.48	-1.66	.18	-1.57
7	10 Apr	3/1-1*	-1.33	-1.49	.16	-1.41
8	10 Apr	3/1-2*	-1.48	-1.66	.18	-1.57
9	11 Apr	1/2-1	-1.39	-1.39	.00	-1.39
10	11 Apr	1/2-2	-1.51	-1.59	.08	-1.55
11	11 Apr	1/3-1	-1.34	-1.47	.13	-1.41
12	11 Apr	1/3-2	-1.46	-1.60	.14	-1.53
13	7 May	1/1	-1.37	-1.48	.11	-1.42
14	7 May	1/2	-1.36	-1.41	.05	-1.38
15	7 May	1/4	-1.35	-1.44	.09	-1.40
16	8 May	3/1	-1.36	-1.46	.10	-1.41
17	8 May	3/2	-1.34	-1.49	.15	-1.42

* Check Runs

TABLE 20

DIRECTION OF GUN OVER THE RANGE
AT THE MOMENT THE GUN WAS FIRED

Date	Mission	Pass	Round	GRADS A	ν	α	ω
21 Mar 58	1	2	1	392.452	198.337	198.349	399.803
21 Mar 58	1	2	1*	392.205	198.325	198.338	399.795
21 Mar 58	2	1	1	8.352	198.302	198.316	.222
28 Mar 58	2	2	1	37.623	196.067	196.733	2.190
10 Apr 58	3	1	1	348.102	197.289	198.140	398.027
10 Apr 58	3	1	1*	348.048	197.281	198.137	398.020
10 Apr 58	3	1	2	26.733	198.410	198.548	.648
10 Apr 58	3	1	2*	26.550	198.398	198.535	.649
11 Apr 58	1	2	1	33.910	196.187	196.714	1.936
11 Apr 58	1	2	2	4.845	196.758	196.767	.246
11 Apr 58	1	3	1	387.775	196.842	196.900	399.398
11 Apr 58	1	3	2	372.214	197.406	197.649	398.904
7 May 58	1	1	1	343.204	196.676	197.912	397.413
7 May 58	1	2	1	39.296	197.070	197.610	1.695
7 May 58	1	4	1	16.491	197.231	197.323	.709
8 May 58	3	1	1	56.150	196.385	197.701	2.790
8 May 58	3	2	1	39.594	197.859	198.260	1.247

* Check Run

TABLE 21

GEOGRAPHIC POSITIONS OF EGLIN AFB RANGE LIGHTS

Point No.	ϕ			λ			H (meters)
Pi C	30°	33'	00".502	86°	37'	19".432	57.362
Pi D	30	37	57.288	86	41	02.675	42.654
MID	30	35	01.732	86	39	59.275	52.6
Coupland	30	35	29.220	86	39	10.464	67.132
9	30	34	50.65184	86	45	29.13569	54.589
12	30	30	44.55020	86	42	16.67930	35.826
13	30	35	44.44512	86	43	54.97437	57.994
16	30	31	33.41819	86	40	44.46125	38.555
17	30	36	38.08818	86	42	24.62201	52.734
20	30	32	35.78161	86	39	13.30891	48.590
21	30	37	30.49215	86	40	42.14681	47.789
24	30	33	24.86331	86	37	38.03528	53.409
25	30	38	26.67464	86	39	08.93853	39.923
28	30	34	18.27018	86	36	01.58315	53.995

TABLE 22

REDUCED COORDINATES OF EGLIN AFB RANGE LIGHTS

	Meters		
Point No.	X	Y	Z
Pi C	-1535.0080	-5452.0527	54.8438
Pi D	-1533.7141	5452.1625	40.1300
MID	0.0000	0.0000	52.600
Coupland	-1551.7021	0.2159	66.9430
9	7550.7517	4511.0490	48.5297
12	7390.1380	4638.9399	29.8382
13	4544.5022	4528.9747	54.7692
16	4508.6016	-4719.4158	35.2021
17	1626.1132	4599.2995	50.8666
20	1424.5047	-4435.2196	46.8823
21	-1541.7195	4462.3607	46.0349
24	-1528.2878	-4552.8420	51.5990
25	-4565.8482	4558.5497	36.6468
28	-4579.7276	-4575.0096	50.7122

TABLE 23

Pi C - ASKANIA CAMERA ORIENTATIONS**
(enforcing X_o Y_o Z_o)

Date	Mission Pass	No. of Stars	m.e. Single Plate Meas. (microns)	α_g m.e. α_g	ω_g m.e. ω_g	κ_g m.e. κ_g	c m.e. of c	x_p m.e. x_p	y_p m.e. of y_p
21 Mar	M1/P2	11	5.01	23.590 .035	-30.677 .018	-55.864 .016	.370482 10.34	.167405 163.68	.190811 129.36
21 Mar	M1/P2*	12	3.25	23.626 .022	-30.675 .011	-56.173 .010	.370475 6.68	.164595 102.968	.190592 82.292
21 Mar	M2/P1	15	5.18	23.618 .017	-30.547 .017	-55.394 .008	.370521 9.24	.183943 108.98	.193334 78.08
28 Mar	M2/P2	17	5.77	23.634 .019	-30.569 .020	-55.880 .009	.370460 11.27	.167160 132.14	.190675 77.55
10 Apr	M3/P1	15	5.35	19.416 .022	-33.159 .028	-64.670 .011	.370511 10.93	.168534 174.09	.192291 95.36
10 Apr	M3/P1*	15	5.38	19.421 .018	-33.163 .026	-64.337 .009	.370506 10.78	.164543 151.64	.189922 94.12
11 Apr	M1/P2	12	4.08	19.446 .024	-33.334 .020	-64.666 .012	.370532 9.46	.164670 149.30	.190245 82.79
11 Apr	M1/P3	14	5.40	19.525 .032	-33.207 .023	-64.098 .016	.370555 10.57	.164695 185.48	.190420 102.47

TABLE 23 (continued)

Pi C - ASKANIA CAMERA ORIENTATIONS
(enforcing X_o Y_o Z_o)

Date	Mission Pass	No. of Stars	m.e. Single Plate Meas. (microns)	α_g m.e. α_g	ω_g m.e. ω_g	κ_g m.e. κ_g	c m.e. of c	x_p m.e. x_p	y_p m.e. of y_p
7 May	M1/P1	11	4.98	24.374 .023	-35.260 .028	-57.608 .012	.370507 11.57	.166636 155.38	.195186 122.67
7 May	M1/P2	6	9.80	24.340 .061	-35.245 .074	-57.353 .032	.370553 30.04	.065545 448.78	.093601 285.35
7 May	M1/P4	8	6.26	24.406 .025	-35.167 .028	-56.008 .013	.370541 14.17	.064253 168.89	.094892 120.94
8 May	M3/P1	14	4.41	24.428 .027	-35.263 .020	-59.082 .014	.370477 10.18	.166574 139.67	.189841 114.70
8 May	M3/P2	7	8.60	24.296 .056	-35.172 .046	-57.133 .030	.370493 24.22	.062757 328.21	.090008 214.04

* Check Runs

** x_p and y_p values are not reduced to a common zero.

c, x_p and y_p given in meters

m.e. of c, x_p and y_p given in microns

TABLE 24

Pi D - ASKANIA CAMERA ORIENTATIONS**
(enforcing X_o Y_o Z_o)

Date	Mission Pass	No. of Stars	m.e. Single Plate Meas. (microns)	α_g m.e. α_g	ω_g m.e. ω_g	κ_g m.e. κ_g	c m.e. of c	x_p m.e. x_p	y_p m.e. y_p
21 Mar	M1/P2	10	4.73	-37.355 .032	6.839 .028	189.818 .004	.370485 14.18	.166764 183.80	.191342 166.22
21 Mar	M1/P2*	11	4.72	-37.375 .028	6.856 .027	189.256 .004	.370466 13.08	.165564 164.79	.189255 161.64
21 Mar	M2/P1	10	5.60	-37.533 .033	6.914 .026	190.216 .004	.370340 14.42	.184110 195.31	.200593 147.74
28 Mar	M2/P2	11	5.04	-37.583 .034	6.853 .022	190.402 .004	.370292 13.98	.170035 202.21	.193083 121.82
10 Apr	M3/P1	15	4.41	-37.939 .023	2.328 .014	195.917 .002	.370330 9.95	.166935 134.61	.190575 80.93
10 Apr	M3/P1*	15	5.73	-37.919 .030	2.310 .018	195.839 .002	.370343 12.87	.165784 174.60	.188454 105.10
11 Apr	M1/P2	11	5.22	-37.968 .029	2.238 .021	196.684 .002	.370446 13.24	.163428 166.82	.191018 126.48
11 Apr	M1/P3	13	5.04	-37.971 .025	2.326 .017	195.587 .002	.370336 10.44	.165162 147.76	.189635 98.88

TABLE 24 (continued)

Pi D - ASKANIA CAMERA ORIENTATIONS**
(enforcing X_o Y_o Z_o)

Date	Mission Pass	No. of Stars	m.e. Single Plate Meas. (microns)	α_g / m.e. α_g	ω_g / m.e. ω_g	κ_g / m.e. κ_g	c / m.e. of c	x_p / m.e. x_p	y_p / m.e. y_p
7 May	M1/P1	7	6.63	-41.756	4.960	194.397	.370356	.079606	.097484
				.041	.028	.004	20.62	232.21	174.17
7 May	M1/P2	7	8.58	-41.726	4.970	194.898	.370350	.063210	.095390
				.051	.037	.005	25.39	304.10	205.30
7 May	M1/P4	7	8.51	-41.810	5.020	195.526	.370300	.065602	.094690
				.042	.034	.005	21.16	244.59	199.07
8 May	M3/P1	13	3.52	-41.850	5.022	192.753	.370343	.164199	.188992
				.016	.013	.002	7.46	93.57	72.30
8 May	M3/P2	7	7.22	-41.775	4.855	194.804	.370421	.062010	.088861
				.031	.028	.004	17.16	182.65	159.83

* Check Runs

** x_p and y_p values are not reduced to a common zero.

c x_p y_p given in meters

m.e. of c x_p and y_p given in microns

TABLE 25

RELATIVE TIMES OF EVENTS
(SECONDS)

	21 Mar M1/P2	21 Mar M2/P1	28 Mar M2/P2	10 Apr M3/P1-R1	11 Apr M1/P2-R1	11 Apr M1/P3-R1	7 May M1/P1	7 May M1/P2	7 May M1/P4	8 May M3/P1	8 May M3/P2
RC-7 Flash	.013262	- .038453	- .137893	- .007635	.499257	- .049790	- .000217	- .047842	- .006869	- .003320	- .036495
Gun Trigger Pulse	.014500	- .037300	- .135300	- .005100	.501050	- .047750	.001500	- .047250	- .171900	- .002200	- .035640
Gun Fire	.016600	- .035200	- .133200	- .003000	.503150	- .045650	.003600	- .045150	-	- .000100	- .033540
Start of Doppler	.019500	- .032500	- .130000	.000120	.506300	- .042350	.006900	- .041800	-	.003350	- .030160
1st Micro	.019904	- .031922	- .130627	.000044	.506291	- .042585	.007910	- .039558	.002180	.004570	- .028076
2nd Micro	.023506	- .028261	- .126950	.003712	.509974	- .038912	.012104	- .035329	.006421	.008707	- .023875
Burst	.875700	.937420	None	None	None	None	None	None	None	None	None
Reset	.1237	.1254	.1366	.14706	.13755	.14681 .	.15471	.165230	.177457	.18774	.189060
					1.43001*						
Post Survey 1	1.9760	1.7830	2.2740	2.31585	1.79888	1.79704	2.80550	2.81670	2.82743	1.82113	2.01340
2	2.4767	2.2838	2.7746	2.80697	2.29808	2.29766	None	-	3.32809	2.30644	2.83440
3	2.9767	2.7838	3.2746	3.30693	2.79807	2.79764	-	-	3.82808	2.83300	3.33482
4	3.4766	3.2838	3.7746	3.80700	3.29798	3.29757	-	-	4.32810	3.33254	3.83486
5	3.9766	3.7838	4.2746	4.30697	3.79801	3.79759	-	-	4.82802	3.83351	4.33476
6	-	-	-	4.80701	4.29803	4.29764	-			4.33353	
7	-	-	-	5.30698	4.79801	4.79762	-			4.83353	
8	-	-	-	5.80696	5.29800	5.29759	-				
9	-	-	-	6.30697	5.79798	5.79760	-				
10	-	-	-	None	6.29803	6.29760	-				
SECOND ROUND				Rd. 2	Rd. 2	Rd. 2					
RC-7 Flash				8.772934	6.949830	6.897346					
Gun Trigger Pulse				8.776300	6.951600	6.899450					
Gun Fire				8.778400	6.953700	6.901550					
Start of Doppler				8.781600	6.956900	6.904950					
1st Micro				8.779677	6.957083	6.904647					
2nd Micro				8.784340	6.960735	6.908320					
Burst				9.735400	None	None					
Reset				7.64680	7.14775	7.14774					
					7.60790*						
					8.79746*						
Post Survey 1				9.30636	9.30017	8.79701					
2				9.81540	9.79809	9.29758					
3				10.30689	10.29804	9.79765					
4				10.80703	10.79803	10.29760					
5				11.30705	11.29812	10.79761					
6				11.80696	11.79808	11.29758					

*Extra Flashes

TABLE 26

ASKANIA RESULTS*
(meters)

	21 March M1/P2 Coordinates			21 March M1/P2 Check Run Coordinates		
	X	Y	Z	X	Y	Z
Burst	2033.94	-636.44	8869.91	2033.89	-636.56	8870.17
Flash RC-7	1839.72	-614.45	9602.65	1839.67	-614.48	9602.30
Reset	1888.94	-615.67	9602.59	1888.77	-615.74	9602.09
Post Survey 1	2285.41	-627.84	9603.90	2285.06	-627.70	9603.27
2	2405.17	-631.57	9604.53	2404.99	-631.46	9604.16
3	2524.74	-635.37	9604.54	2524.63	-635.18	9604.38
4	2644.17	-639.21	9604.84	2643.96	-639.00	9604.29
5	2763.43	-643.19	9605.22	2763.31	-643.03	9604.86

	21 March M2/P1 Coordinates			28 March M2/P2 Coordinates		
	X	Y	Z	X	Y	Z
Burst	2256.41	-501.91	8654.97	1956.55	-204.68	8841.73
Flash RC-7	2049.08	-499.04	9486.39	1762.56	-206.60	9593.37
Reset	2086.57	-496.46	9485.81	1820.67	-209.56	9595.13
Post Survey 1	2474.62	-476.87	9488.42	2274.57	-234.58	9609.97
2	2591.60	-470.84	9488.97	2380.01	-240.47	9613.38
3	2708.69	-464.94	9489.94	2485.50	-246.50	9617.18
4	2825.53	-459.38	9490.05	2590.52	-252.47	9620.51
5	2942.65	-453.54	9490.93	2695.82	-258.42	9624.11
				2800.82	-264.59	9627.60

	10 Apr M3/P1 Rd. 1 Coordinates			10 Apr M3/P1 Rd. 1 Check Run Coordinates		
	X	Y	Z	X	Y	Z
1st Burst	1955.32	-737.25	8864.11	1955.40	-737.25	8864.14
Flash RC-7	1775.00	-688.40	9664.42	1775.13	-688.47	9664.30
Reset	1805.85	-689.72	9664.29	1805.90	-689.92	9664.33
Post Survey 1	2242.42	-711.26	9667.10	2242.42	-711.50	9667.15
2	2341.22	-716.29	9667.40	2341.27	-716.41	9667.40
3	2442.26	-721.38	9668.32	2442.07	-721.52	9667.81
4	2542.93	-726.57	9668.19	2542.66	-726.61	9668.13
5	2643.85	-731.71	9668.57	2643.72	-731.80	9668.58
6	2744.53	-737.04	9668.72	2744.41	-737.19	9668.44
7	2845.53	-742.37	9668.62	2845.31	-742.60	9668.42
8	2946.20	-747.99	9668.40	2946.01	-747.99	9668.22
9	3047.24	-753.62	9668.00	3047.10	-753.85	9667.99

TABLE 26 (continued)
ASKANIA RESULTS*
(meters)

	10 Apr M3/P1 Rd. 2 Coordinates			10 Apr M3/P1 Rd. 2 Check Run Coordinates		
	X	Y	Z	X	Y	Z
2nd Burst	3720.04	-801.11	8870.15	3719.78	-801.22	8869.53
Flash RC-7	3545.90	-782.67	9668.28	3545.65	-782.94	9667.98
Reset	3317.84	-769.14	9668.08	3318.93	-770.52	9668.78
Post Survey 1	3653.06	-788.92	9667.87	3653.02	-789.27	9667.75
2	3755.88	-795.26	9668.16	3755.80	-795.46	9667.82
3	3855.50	-801.56	9668.27	3855.42	-801.73	9668.00
4	3956.13	-807.81	9668.45	3956.18	-808.00	9668.28

	11 Apr M1/P2 Rd. 1 Coordinates			11 Apr M1/P3 Rd. 1 Coordinates		
	X	Y	Z	X	Y	Z
1st Burst	2597.11	-262.29	7947.88	2012.21	-573.79	8807.13
RC-7 Flash	2237.76	-275.77	9550.79	1816.53	-552.96	9643.26
Reset	2161.39	-271.75	9550.09	1858.13	-554.75	9643.74
Post Survey 1	2434.38	-286.08	9553.77	2206.09	-571.11	9645.13
2	2512.07	-290.32	9554.67	2311.84	-576.26	9645.54
3	2617.54	-296.25	9555.77	2417.55	-581.76	9645.30
4	2723.28	-302.42	9557.34	2523.10	-586.86	9645.11
5	2828.87	-308.78	9558.40	2629.00	-592.18	9644.44
6	2934.50	-315.01	9559.43	2734.69	-597.72	9644.08
7	3040.27	-321.34	9560.61	2840.51	-603.20	9643.90
8	3146.14	-327.96	9561.25	2946.16	-608.68	9642.79
9	3252.07	-334.68	9562.42	3052.26	-614.33	9641.99
10	3357.98	-341.46	9563.49	3157.97	-619.88	9641.44
11	3463.95	-348.32	9564.86			

	11 Apr M1/P2 Rd. 2 Coordinates			11 Apr M1/P3 Rd. 2 Coordinates		
	X	Y	Z	X	Y	Z
2nd Burst	3785.27	-370.56	8789.41	3469.42	-655.88	8820.55
RC-7 Flash	3602.21	-357.40	9566.35	3284.94	-626.96	9640.21
Reset	3644.18	-360.20	9566.56	3338.14	-630.01	9639.69
Post Survey 1	4100.21	-391.41	9570.74	NOT MEASURABLE		
2				3794.08	-655.53	9634.50
3				3900.33	-661.76	9633.14
4				4006.41	-667.59	9631.45

TABLE 26 (continued)

ASKANIA RESULTS*
(meters)

	7 May M1/P1 Rd. 1 Coordinates			7 May M1/P2 Rd. 1 Coordinates		
	X	Y	Z	X	Y	Z
Burst	2424.41	-600.48	8043.85	2206.63	-206.72	7948.65
RC-7 Flash	1954.48	-488.47	9692.68	1745.88	-217.86	9539.18
Reset	1993.77	-488.12	9692.96	1798.58	-216.81	9540.21
Post Survey 1	2658.53	-480.31	9697.08	2450.35	-203.08	9554.32

	7 May M1/P4 Rd. 1 Coordinates		
	X	Y	Z
Burst	2259.18	-403.84	8035.36
RC-7 Flash	1787.59	-395.14	9622.09
Reset	1832.61	-393.73	9623.03
Post Survey 1	2479.39	-372.94	9639.60
2	2601.78	-369.01	9642.74
3	2723.76	-365.44	9646.26
4	2846.22	-361.70	9650.70
5	2968.18	-358.04	9654.19

	8 May M3/P1 Rd. 1 Coordinates			8 May M3/P2 Rd. 1 Coordinates		
	X	Y	Z	X	Y	Z
Burst	2226.01	-429.83	8125.42	2247.74	-642.51	8086.27
RC-7 Flash	1805.07	-461.06	9702.40	1827.49	-646.74	9690.31
Reset	1850.03	-462.07	9702.76	1880.32	-646.77	9690.66
Post Survey 1	2234.35	-469.56	9709.15	2311.04	-648.10	9693.02
2	NOT MEASURED			2503.59	-649.37	9693.55
3	2472.83	-475.09	9713.90	2621.73	-650.31	9693.99
4	2590.59	-478.30	9716.21	2739.57	-651.28	9693.91
5	2708.44	-481.70	9718.81	2857.89	-652.84	9693.50
6	2826.21	-485.30	9721.42			
7	2944.04	-489.20	9724.10			

* Z Coordinates not corrected for refraction

TABLE 27

CROSSWIND RESULTS

(a) SPACE COORDINATES OF AIRCRAFT AT TIME OF FIRING:

1. Firing - Coordinates listed are for the position of the end of the gun barrel in meters at time of firing

2. Burst - Coordinates listed are for position of burst in meters

(b) AVERAGE VELOCITY OF AIRCRAFT: given in meters/sec

(c) ORIENTATION OF THE GUN CENTER LINE IN SPACE AT THE MOMENT OF FIRING

(d) MUZZLE VELOCITY:

Muzzle velocity of the projectile as obtained by optical and electronic methods, 17.5 ft/sec constant bias not applied to electronic method

(e) TIME EMERGENCE OF PROJECTILE:

Elapsed time in seconds between time gun fires and projectile emerges from barrel

(f) TIME OF BURST OF PROJECTILE:

Elapsed time in seconds between time gun fires and burst occurs

Date		(a)			(b)	(c)		(d)		(e)	(f)
Miss/Pass Round		X m	Y	Z	m/sec	α^g / A^g	ω^g / ν^g	Optics m/sec	Elect m/sec	Emerge sec	Burst sec
21 Mar	1	1840.14	-614.45	9601.44	239.06	198.35	399.80	None	970.79	.002900	.859100
M1/P2	2	2033.94	-636.44	8868.61		392.45	198.34				
21 Mar	1	2049.46	-499.04	9485.18	234.26	198.32	.222	None	999.14	.002700	.972620
M2/P1	2	2256.41	-501.91	8653.69		8.35	198.30				
28 Mar	1	1763.17	-206.60	9592.17	211.02	196.73	2.19	None	986.94	.003200	None
M2/P2	2	1956.55	-204.68	8840.45		37.62	196.07				
10 Apr	1	1775.51	-688.40	9663.20	201.66	198.14	398.03	None	957.68	.003120	None
M3/P1-1	2	1955.32	-737.25	8862.82		348.10	197.29				
10 Apr	1	3546.62	-782.67	9666.90	202.23	198.55	.65	None	964.08	.003200	.957000
M3/P1-2	2	3720.04	-801.11	8868.69		26.73	198.41				
11 Apr	1	2238.20	-275.77	9549.59	211.81	196.71	1.94	None	956.16	.003150	None
M1/P2-1	2	2597.11	-262.29	7946.68		33.91	196.19				
11 Apr	1	3602.65	-357.40	9564.99	212.30	196.77	.25	None	955.55	.003200	None
M1/P2-2	2	3785.27	-370.56	8787.96		4.84	196.76				
11 Apr	1	1817.03	-552.96	9642.04	211.71	196.90	399.40	None	951.59	.003300	None
M1/P3-1	2	2012.21	-573.79	8805.85		387.78	196.84				
11 Apr	1	3285.46	-626.96	9638.87	212.60	197.65	398.90	None	938.18	.003400	None
M1/P3-2	2	3469.42	-655.88	8819.13		372.21	197.41				
7 May	1	1955.06	-488.47	9691.45	250.80	197.91	397.41	977.09	972.01	.003300	None
M1/P1	2	2424.41	-600.48	8042.57		343.20	196.68				
7 May	1	1746.16	-217.86	9537.99	245.58	197.61	1.70	957.68	954.64	.003350	None
M1/P2	2	2206.63	-206.72	7947.48		39.30	197.07				
7 May	1	1747.37	-395.14	9620.88	244.49	197.32	.71	946.10	None	None	None
M1/P4	2	2259.18	-403.84	8034.17		16.49	197.23				
7 May		Missing flashes on Askania plates						940.61	932.38		
M1/P5											
8 May	1	1805.45	-461.06	9701.18	235.66	197.70	2.79	None	966.22	.003450	None
M3/P1	2	2226.01	-429.83	8124.22		56.15	196.38				
8 May	1	1827.80	-646.74	9689.09	235.78	198.26	1.25	961.95	956.77	.003380	None
M3/P2	2	2247.74	-642.51	8085.07		39.59	197.86				

TABLE 28

PROJECTILE VELOCITIES

(by Velocity Cameras and Corresponding P.V.M. Results)

	7 May				8 May
	M1/P1	M1/P2	M1/P4	M1/P5	M3/P2
Projectile @t_2					
X	97.4531	97.4585	97.4536	97.4626	97.4298
Y	96.6931	96.6921	96.6923	96.6908	96.7140
Z	- .0910	- .0536	- .0142	- .1343	- .1588
Projectile @t_1					
X	97.3827	97.3829	97.3829	97.3856	97.3608
Y	96.7023	96.7024	96.7024	96.7030	96.7006
Z	4.0063	3.9960	3.9973	3.8456	3.8822
ΔX	.0704	.0756	.0707	.0770	.0690
ΔY	- .0092	- .0103	- .0101	- .0122	.0134
ΔZ	- 4.0973	- 4.0496	- 4.0115	- 3.9799	- 4.0410
Distance	4.0979	4.0503	4.0121	3.9807	4.0416
Time (sec)	.004194	.004229	.004241	.004232	.004201
Velocity meters/sec	977.1	957.7	946.1	940.6	962.0
Velocity ft/sec	3205	3142	3140	3086	3156
Electronic (P.V.M. System)	3189	3132	None	3059	3139
Diff. (ft/sec)	16	10		27	17

TABLE 29

Date	PVM Value	Final Projectile Velocity ft/sec
21 March 1/2	3185	3202
21 March 2/1	3278	3295
28 March 2/2	3238	3255
10 April 3/1-1	3142	3159
10 April 3/1-2	3163	3180
11 April 1/2-1	3137	3154
11 April 1/2-2	3135	3152
11 April 1/3-1	3122	3139
11 April 1/3-2	3078	3095
7 May 1/1	3189	3206
7 May 1/2	3132	3149
7 May 1/5	3059	3076
8 May 3/1	3170	3187
8 May 3/2	3139	3156

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